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ABSTRACT

The objective of the Reactor Dynamics Module, RD-1, is to obtain the kinetics equation without feedback and solve the kinetics equations numerically for one to six delayed neutron groups for time varying reactivity insertions. The computer code FUMCKI (Fundamental Mode Kinetics) will calculate the power as a function of time for either uranium or plutonium. Either fuel can be used with one to six delayed neutron groups and one of three types of reactivity insertions: a constant reactivity, sinusoidal, or a ramp. The code does not compute any parameters so the neutron generation time must be provided. The user has the option of studying the effects of various time steps in solving the system. The objective of Module RD-2 is to examine the temperature feedback mechanism of a pressurized water reactor (PWR) and solve the one delayed neutron model with temperature feedback for a step insertion and a ramp insertion of reactivity. A PWR core with a two-path feedback is considered. The core region is the only one of interest in this module. The program name is FUMOTEM (Fundamental Mode Kinetics with Temperature Feedback). There are four types of reactivity inputs that the program can accommodate. (Author)

REACTOR DYNAMICS MODULE, RD-1

THE REACTOR KINETICS EQUATIONS

by

Ronald J. Onega

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KINETICS MODULE 1

THE REACTOR KINETICS EQUATIONS

1.1 Object of Module

The object of this module is to:

1) Obtain the Kinetics Equation without feedback

and 2) Solve the kinetics equations numerically for one to six delayed neutron groups for time varying reactivity insertions.

The time dependence of a modern reactor is really very complicated. The control rod motion is a local perturbation so the time dependence of the flux cannot completely be divorced from the space dependence. The fundamental mode kinetics equations do provide a rough idea of what the time behavior of a reactor will be. In this module we will develop the kinetics equations and indicate how they can be solved numerically. Feedback effects will be introduced in later modules.

The computer code FUMOKI (Fundamental Mode Kinetics) will calculate the power as a function of time for either uranium or plutonium. Either fuel can be used with one to six delayed neutron groups and one of three types of reactivity insertions:

1) a constant reactivity ρ_o

2) sinusoidal $\rho(t) = \rho_o \sin b_2 t$

or 3) a ramp $\rho(t) = \rho_o(1 + b_3 t)$.

The ramp is to simulate the rod withdrawl or insertion reactivity input.

The code does not compute any parameters so the neutron generation time must be provided. Also the user has the option of studying the effects of various time steps in solving the system. The time step can be surprisingly large (0.05 sec) and still yield good results in most cases.

1.2 The Kinetics Equations

The time behavior of a reactor is a very important consideration in the operation of a nuclear power plant. Also, the safety analysis of a plant depends upon a thorough knowledge of the kinetics equations. The many types of reactor designs necessitates consideration of various reactivity coefficients and dynamic response characteristics.

The neutronic considerations of a reactor cannot be divorced from associated feedback mechanisms such as heat transport, fluid flow, mechanical changes etc. There are many ways to delineate the dynamics problems of a power plant but a natural one seems to be to classify problems according to the time constants involved. There are four areas that we can study:

1) Very slow transients - fuel depletion with time constants of the order of a year or so. We will not consider this as a dynamics problem at all but rather in the statics sections.
2) Slow transients - Xenon and Samarium effects. The time constants here are of the order of hours.
3) Normal transients - Changes in fuel and moderator temperature, void changes, delayed neutron considerations etc. The time constants here are of the order of a second or so and small reactivity changes are involved.
4) Fast transients - Control rod dropped in or withdrawn at its maximum rate etc. Reactivity inputs are of 50¢ and up. The time constants are of the order of 10^{-4} sec. and serious safety questions are raised.

In addition to these rather general time reference frames, another important aspect of the reactor dynamics problem is whether or not the core is so large that spatially dependent analysis is necessary. The solution of one or more dimensional kinetics problems necessitates the use of a relatively large computer. Xenon oscillations are of importance here. Also reactivity insertions are usually localized so that hot spots may develop.

Coupled core kinetics considerations are of interest in some types of reactors with large cores or core regions which are loosely coupled. When the neutron flight times between different regions of the reactor are not negligible, then coupled core kinetics may be a useful tool. Coupled core kinetics equations are generally differential-integral equations with time-lag kernels. Coupled core kinetics involves writing the kinetics equations for each region of the reactor and then coupling the regions by neutron leakage from one region to the other.

When a control rod in a reactor is removed, the neutron flux distribution is disturbed so that not only the fundamental mode is present but the higher modes are also present. However, these higher harmonics die out very rapidly so that while the rod is in motion (in the order of seconds) the fundamental mode is the only important one. Therefore we will deal only with the fundamental mode for the space dependence and the kinetics equations we solve will simply indicate how the amplitude of this fundamental mode changes with time. For example, in a spherical reactor the flux is

$$\phi(r,t) = \phi_0(t)\,\frac{\sin Br}{Br} ,$$

and this spatial dependence persists through most transients, and the kinetics equations simply yield the $\phi_o(t)$. In a word, the spatial and time dependence are separable for most transients.

We will derive the kinetics equations using one group diffusion theory but the same equations are obtained from multi-group or transport theory. The thermal neutron diffusion equation is

$$D \nabla^2 \phi - \Sigma_a \phi + S = \frac{1}{v} \frac{\partial \phi}{\partial t} \quad . \qquad (1.2.1)$$

The source is composed of three terms, i.e.

$$S = S_p + S_D + S_{ext} \quad , \qquad (1.2.2)$$

where S_p = prompt neutron source,

S_D = delayed neutron source,

and S_{ext} = a source of neutrons entered externally, as from a Plutonium-Beryllium source.

Not all neutrons are emitted immediately from the fission process. The prompt neutrons are emitted within 10^{-10} sec. of the fission process itself but there are other neutrons called delayed neutrons. They arise from the beta decay of some of the fission fragments. As an example, Kr-93 is a fission fragment which has a half life of 1.22 sec. This beta-decays to Rb-93, but it is formed in such a highly excited state that a neutron is emitted following the beta decay and the reaction is thus,

$$^{93}Kr \xrightarrow[1.22 \text{ sec.}]{\beta} {}^{93}Rb \xrightarrow[\sim 10^{-16} \text{ sec.}]{} {}^{92}Rb + {}^{1}_{o}n \; .$$

There are some 30 of these neutron precursors (^{93}Kr) which produce neutrons

through beta decay.

Since the half-lives of many of these precursors are very close to each other, it is necessary only to consider six delayed neutron precursor groups which are averages of the 30 or so precursors, appropriately weighted. The delayed neutron precursors we talk about are thus fictitious in that they do not actually exist, but are averages of the actual precursors. Table 1.5.1 (section 5) indicates the groups of delayed neutrons obtained from these six precursors for neutron induced fission in uranium and plutonium.

The fraction of neutrons which are delayed is called β. For ^{235}U, $\beta = 0.0064$ and the total delayed neutron fraction is the sum of the individual ones, i.e.,

$$\beta = \sum_{i=1}^{6} \beta_i. \qquad (1.2.3)$$

Returning now to the diffusion equation, we can set

$$S_p = (1-\beta)\, \nu\Sigma_f\, \phi(\vec{r}, t) \qquad (1.2.4)$$

and

$$S_D = \sum_{i=1}^{6} \lambda_i\, C_i\, (\vec{r}, t) \qquad (1.2.5)$$

where $C_i(\vec{r}, t)$ is the precursor concentration of group "i", i.e., the number of precursors/cm^3 existing at point $\vec{r}$ at time t. Each time a precursor C_i decays, it is assumed a delayed neutron of group "i" results.

So our diffusion equation becomes (Equation 1.2.1)

$$D\nabla^2 \phi - \Sigma_a \phi + (1-\beta)\, \nu\Sigma_f\, \phi + \sum_{i=1}^{6} \lambda_i\, C_i\, (\vec{r}, t) = \frac{1}{v}\frac{\partial\phi}{\partial t} \qquad (1.2.6)$$

Equation (1.2.6) has four independent variables, x, y, z and t, and seven dependent variables ϕ, C_1, C_2, ..., C_6. In order to solve such a system of equations, we will have to write six more equations, one for each of the precursor concentrations. The rate at which the precursor concentration changes is

$$\frac{\partial C_i(\vec{r}, t)}{\partial t} = \underbrace{\beta_i \nu\Sigma_f \phi(\vec{r}, t)}_{\text{rate of formation}} - \underbrace{\lambda_i C_i(\vec{r}, t)}_{\text{rate of decay}}. \qquad (1.2.7)$$

$$i = 1, 2, \ldots, 6.$$

This system of equations is relatively difficult to solve so we shall work analytically with Equation (1.2.6) and (1.2.7) for a while.

Assume that we want to expand the spatial dependence of the flux $\phi(\vec{r}, t)$ and the precursor concentrations $C_i(\vec{r}, t)$ in terms of a set of eigenfunctions of the Helmholtz equation

$$\nabla^2 Y_n(\vec{r}) + B_n^2 Y_n(\vec{r}) = 0. \qquad (1.2.8)$$

The reason for doing this is that the solution for the steady state satisfies this same equation with coefficients which are time independent. In the time dependent problems the coefficients are functions of time. The eigenfunctions $Y_n(\vec{r})$ are Cos B_n x for a slab reactor, Bessel's functions for a cylindrical reactor, etc. In any event, we set

$$\phi(\vec{r}, t) = \sum_{n=0}^{\infty} \phi_n(t) Y_n(\vec{r}) \qquad (1.2.9)$$

and

$$C_i(\vec{r}, t) = \sum_{n=0}^{\infty} \tilde{C}_{in}(t) \, Y_n(\vec{r}). \qquad (1.2.10)$$

We note that $\tilde{\phi}_n(t)$ will yield the amplitude of the $n^{\underline{th}}$ harmonic and $Y_n(\vec{r})$ its spatial distribution. Substituting these expressions into Equations (1.2.6) and (1.2.7) and using Equation (1.2.8) we obtain

$$\sum_{n=0}^{\infty} \left[-DB_n^2 \, \tilde{\phi}_n(t) - \Sigma_a \tilde{\phi}_n(t) + (1-\beta) \, \nu\Sigma_f \, \tilde{\phi}_n(t) \right] Y_n(\vec{r}) + \sum_{i=1}^{6} \sum_{n=0}^{\infty} \lambda_i \, \tilde{C}_{in}(t) \, Y_n(\vec{r}) = \frac{1}{v} \sum_{n=0}^{\infty} Y_n(\vec{r}) \, \frac{d\tilde{\phi}_n(t)}{dt}$$

and

$$\sum_{n=0}^{\infty} Y_n(\vec{r}) \, \frac{d\tilde{C}_{in}(t)}{dt} = \sum_{n=0}^{\infty} \left[\beta_i \, \nu\Sigma_f \, \tilde{\phi}_n(t) - \lambda_i \, \tilde{C}_{in}(t) \right] Y_n(\vec{r}).$$

Now using the fact that the $Y_n(\vec{r})$ form an orthogonal set, the preceding equation is simplified by multiplying it by $Y_m(\vec{r})$ and integrating over the reactor volume (taking the inner product) to yield

$$\frac{1}{v} \frac{d\tilde{\phi}_n(t)}{dt} = -(DB_n^2 + \Sigma_a) \, \tilde{\phi}_n(t) + (1-\beta) \, \nu\Sigma_f \, \tilde{\phi}_n(t) + \sum_{i=1}^{6} \lambda_i \, \tilde{C}_{in}(t), \qquad (1.2.11)$$

and

$$\frac{d\tilde{C}_{in}(t)}{dt} = \beta_i \, \nu\Sigma_f \, \tilde{\phi}_n(t) - \lambda_i \, \tilde{C}_{in}(t). \qquad (1.2.12)$$

Now it is necessary to solve these two equations for the expansion coefficients $\tilde{\phi}_n(t)$ and $\tilde{C}_{in}(t)$. This is a very difficult task so we

make some definitions and some approximations.

First we define the multiplication k_n as

$$k_n = \frac{\nu\, \Sigma_f/\Sigma_a}{1 + B_n^2 L^2} \tag{1.2.13}$$

and the thermal neutron lifetime

$$\ell_n = \frac{1}{\bar{v}\, \Sigma_a (1 + B_n^2 . L^2)} \tag{1.2.14}$$

The next thing we do is assume that there are no delayed neutrons (there are, but assume for the moment we can neglect them). Then Equation (1.2.11) becomes (if $\beta = 0$),

$$\frac{d\tilde{\phi}_n}{dt}(t) = -v\, \Sigma_a (1 + L^2 B_n^2)\, \tilde{\phi}_n(t) + v\, \nu\Sigma_f\, \tilde{\phi}_n(t).$$

Now using Equation (1.2.14) we have

$$\frac{d\tilde{\phi}_n}{dt}(t) = -\frac{\tilde{\phi}_n}{\ell_n}(t) + v\, \nu \frac{\Sigma_f}{\Sigma_{au}} \frac{\Sigma_{au}}{\Sigma_a} \Sigma_a\, \tilde{\phi}_n(t),$$

$$= -\frac{\tilde{\phi}_n(t)}{\ell_n} + \bar{v}\, \Sigma_a\, k_\infty \cdot \frac{1 + B_n^2 L^2}{1 + B_n^2 L^2}\, \phi_n(t),$$

$$= \frac{k_n - 1}{\ell_n}\, \phi_n(t). \tag{1.2.15}$$

The solution of Equation (1.2.15) is obviously

$$\tilde{\phi}_n(t) = \tilde{\phi}_n(o)\, e^{\frac{k_n - 1}{\ell_n} t} \tag{1.2.16}$$

For a typical light water reactor, $\ell_o = \ell = 10^{-4}$ sec. and $k_o = k = 1.0020$. From Equation (1.2.14) we note that

$$\frac{\ell_n}{\ell} = \frac{1 + B_o^2 L^2}{1 + B_n^2 L^2} \sim \frac{1}{n^2}$$

so that the higher harmonics die out rapidly. We will keep only the fundamental mode, not only in the solution of the no delayed neutron case but also for the solution of Equations (1.2.11) and (1.2.12).

Returning now to Equations (1.2.11) and (1.2.12) and omitting the subscript n since we are concerned only with the fundamental mode, we set

$$\tilde{\phi}(t) = v\, n(t)$$

where n (t) is the neutron density (cm^{-3}), and after a bit of algebra come up with

$$\frac{dn(t)}{dt} = (1-\beta)\, \nu \frac{\Sigma_f}{\Sigma_a}\, v\, \Sigma_a\, n(t) - v\, \Sigma_a (1 + B^2 L^2)\, n(t) + \sum_{i=1}^{6} \lambda_i \tilde{C}_i(t)$$

and

$$\frac{d\tilde{C}_i(t)}{dt} = \beta_i\, v\, \Sigma_a\, \nu \frac{\Sigma_f}{\Sigma_a} n(t) - \lambda_i \tilde{C}_i(t).$$

Using the definitions given in Equations (1.2.13) and (1.2.14) the above equations become

$$\frac{dn(t)}{dt} = \cancel{(1-\beta)\frac{k}{\ell} n} - \frac{n}{\ell} + \sum_{i=1}^{6} \lambda_i \tilde{C}_i(t)$$

and

$$\frac{d\tilde{C}_i(t)}{dt} = \beta_i \frac{k}{\ell} n(t) - \lambda_i \tilde{C}_i(t).$$

k is the effective multiplication, and the effective neutron lifetime is denoted as ℓ. If we drop the tilde on the C_i's, then we have

$$\frac{dn(t)}{dt} = \frac{(1-\beta)k-1}{\ell} n(t) + \sum_{i=1}^{6} \lambda_i C_i(t) \qquad (1.2.18)$$

and

$$\frac{dC_i(t)}{dt} = \beta_i \frac{k}{\ell} n(t) - \lambda_i C_i(t). \qquad (1.2.19)$$

These are called the fundamental mode reactor kinetics equations, in that spatial and time dependence are assumed separable.

Sometimes it's convenient to cast these equations into a slightly different form. The thermal neutron lifetime is

$$\ell = \frac{\lambda_a}{v} \frac{1}{1+B^2L^2}, \qquad (1.2.20)$$

i.e., the mean time a neutron spends in the system from its birth as a thermal neutron until it's absorbed or leaks out of the reactor. The neutron gereration time is defined as

$$\Lambda = \frac{\ell}{k} = \frac{1}{v\,\Sigma_a(1+B^2L^2)} \cdot \frac{\Sigma_a(1+B^2L^2)}{v\,\Sigma_f} = \frac{1}{v\,\nu\Sigma_f} \qquad (1.2.21)$$

The generation time is the mean time that it takes one neutron to generate one more prompt neutron or one precursor. The neutron lifetime is thus the reciprocal of the destruction rate of neutrons and the generation time is the reciprocal of the production rate of neutrons. With this definition and the definition of reactivity ρ,

$$\rho \equiv \frac{k-1}{k}\,, \tag{1.2.22}$$

the kinetics equations (1.2.18 and 1.2.19) become

$$\frac{dn}{dt}(t) = \frac{\rho(t) - \beta}{\Lambda}\, n(t) + \sum_{i=1}^{6} \lambda_i\, C_i\,(t) \tag{1.2.23}$$

$$\frac{d\,C_i\,(t)}{dt} = \frac{\beta_i}{\Lambda}\, n(t) - \lambda_i\, C_i\,(t) \tag{1.2.24}$$

$$i = 1,2,\ldots,6$$

Equations (1.2.23) and (1.2.24) are used to describe the time behavior of a reactor. This form of the equations results if multigroup diffusion theory or transport theory is used to derive the equations but the definitions of ℓ, Λ, ρ, k are modified. We will assume they are input parameters to the system of equations.

Problem 1.2.1

Estimate the neutron lifetime and neutron generation time in an infinite stack of graphite and U-235 if $\sigma_a = 5 \times 10^{-3}$ b for the graphite and there are 500 atoms of carbon to each atom of U-235. There is no U-238 present. $\sigma_a = 680$ b for the U-235 and take v to be 2200 m/sec.

Problem 1.2.2

Estimate the period of a reactor if there are no delayed neutrons and if the k = 1.0010 and $\ell = 10^{-4}$ sec. The reactor period is the time it takes for the flux or neutron density to increase by a factor of e.

Problem 1.2.3

Let $Y_i = C_i(t) \dfrac{\Lambda \lambda_i}{\beta_i}$ and $\alpha_R = \beta/\Lambda$. Show that the kinetics equations are then

$$\frac{dn}{dt} = \alpha_R \left[n(t) \quad (\rho'-1) + \sum_{i=1}^{6} a_i Y_i\right]$$

and

$$\frac{dY_i}{dt} = \lambda_i \left[n(t) - Y_i(t)\right]$$

where $\rho' = \dfrac{\rho(t)}{\beta}$ and $a_i = \beta_i/\beta$.

1.3 Analytical Solutions of the Reactor Kinetics Equations

The kinetics equations are relatively difficult to solve both analytically and numerically due to the large difference in the time constants in the equations as well as due to the fact that there are seven coupled ordinary differential equations if there are six delayed neutron groups.

Equations (1.2.23) and (1.2.24) are repeated here as

$$\frac{dn}{dt} = \frac{\rho(t) - \beta}{\Lambda} n(t) + \sum_{i=1}^{6} \lambda_i C_i(t) \tag{1.3.1}$$

and

$$\frac{dC_i(t)}{dt} = \frac{\beta_i n(t)}{\Lambda} - \lambda_i C_i(t) \qquad i = 1, 2, \ldots, 6. \tag{1.3.2}$$

If we assume that we can take an appropriate average for the delayed neutron decay constant, then it is possible to collapse these seven equations into two equations. For one effective group of delayed neutrons, Equations (1.3.1) and (1.3.2) reduce to

$$\frac{dn(t)}{dt} = \frac{\rho(t) - \beta}{\Lambda} n(t) + \lambda C(t) \tag{1.3.3}$$

and

$$\frac{dC(t)}{dt} = \frac{\beta}{\Lambda} n(t) - \lambda C(t). \tag{1.3.4}$$

The decay constant λ for this average delayed neutron precursor $C(t)$ is

$$\frac{1}{\lambda} = \frac{1}{\beta} \sum_{i=1}^{6} \frac{\beta_i}{\lambda_i} \tag{1.3.5}$$

This average is not unique and at times it may be advantageous to use another expression for the average. If very small reactivities are added to the reactor, then instead of averaging over all six groups, perhaps only the longest three would be used.

If only one group is desired Equation (1.3.5) is used to determine the effective λ and β is simply the sum of the β_i. For two groups, we split the β's into two groups of three each and

$$\beta_1 = \sum_{i=1}^{3} \beta_i, \qquad \frac{1}{\lambda_1} = \frac{1}{\beta_1} \sum_{i=1}^{3} \frac{\beta_i}{\lambda_i}, \tag{1.3.5a}$$

and

$$\beta_2 = \sum_{i=4}^{6} \beta_i, \qquad \frac{1}{\lambda_2} = \frac{1}{\beta_2} \sum_{i=4}^{6} \frac{\beta_i}{\lambda_i}. \tag{1.3.5b}$$

For three groups, the β's are split into three groups of two components each and for four groups, the shortest two groups are averaged together as well as the next shortest two and the 22 and 55 sec. groups are treated separately.

We will solve Equations (1.3.3) and (1.3.4) subject to a constant reactivity insertion ρ_o at time zero. In order to do this we recast our equations into a matrix form because of the similarity to the numerical technique (Hansen's method) used in the solution of the equations. In matrix form, Equations (1.3.3) and (1.3.4) become

$$\frac{d\underline{\psi}(t)}{dt} = \underline{\underline{A}}\, \underline{\psi}(t) \tag{1.3.6}$$

with

$$\underline{\psi}(t) = \begin{bmatrix} n\ (t) \\ \\ C\ (t) \end{bmatrix} \quad \text{and} \quad \underline{\underline{A}} = \begin{bmatrix} \frac{\rho_o - \beta}{\Lambda} & \lambda \\ \\ \frac{\beta}{\Lambda} & -\lambda \end{bmatrix}.$$

Notice that the $\underline{\underline{A}}$ matrix is independent of time if the reactivity is a constant. Equation (1.3.6) can formally be integrated to yield

$$\underline{\psi}\ (t) = e^{\underline{\underline{A}}t} \cdot \underline{\psi}\ (0). \tag{1.3.7}$$

The initial condition vector $\underline{\psi}$ (0) is

$$\underline{\psi}\ (0) = \begin{bmatrix} n(0) \\ \\ C(0) \end{bmatrix} = n(0) \begin{bmatrix} 1 \\ \\ \frac{\beta}{\lambda\Lambda} \end{bmatrix} \tag{1.3.8}$$

where we have used Equation (1.3.4) and set the derivative $\frac{dC}{dt} = 0$ for $t \leq 0$. When reactivity is added at time zero, the delayed neutron precursor concentration C(t) does not change until after some time. It is assumed that the reactor has been operating for a long time at a reactor power consistent with a neutron density n(0).

The formal solution given by Equation (1.3.7) is not much good if the explicit functional dependence of n(t) and C(t) cannot be obtained. The reason no explicit functional dependence is achieved from Equation (1.3.7) is that matrix $\underline{\underline{A}}$ is not diagonal. This means we really haven't separated the equations from each other. In order to demonstrate how the solution can be obtained, we imitate the technique used for the solution of an

$n^{\underline{th}}$ order differential equation, i.e. set

$$\underline{\psi}(t) = e^{\omega t}\,\underline{v}\,, \tag{1.3.9}$$

where $\underline{v}$ is a constant vector and ω is a scalar independent of time. Substituting this into Equation (1.3.6) we have

$$\omega\, e^{\omega t}\,\underline{v} = \underline{\underline{A}}\, e^{\omega t}\,\underline{v}$$

or

$$\underline{\underline{A}}\,\underline{v} = \omega\,\underline{v}. \tag{1.3.10}$$

Now it is apparent that Equation (1.3.10) is an eigenvalue equation so

$$|\underline{\underline{A}} - \omega\underline{\underline{I}}| = 0 \tag{1.3.11}$$

must be satisfied to determine the eigenvalue ω. This equation is called the characteristic equation and in reactor dynamics it is called the "in-hour equation" because it relates the reactivity inserted into $\underline{\underline{A}}$ with the ω which is the reciprocal of the reactor period.

The inhour equation (Equation 1.3.11) for the one group delayed neutron model is

$$\det\begin{bmatrix} \frac{\rho_o - \beta}{\Lambda} - \omega & \lambda \\ \frac{\beta}{\Lambda} & -\lambda-\omega \end{bmatrix} = 0 \tag{1.3.12}$$

or

$$\Lambda\,\omega^2 + (\beta - \rho_o + \Lambda\,\omega) - \lambda\,\rho_o = 0. \tag{1.3.13}$$

Equation (1.3.13) can also be written as

$$\rho_o = \Lambda\, \omega + \frac{\beta\omega}{\omega+\lambda} . \tag{1.3.14}$$

If we had kept all six groups of delayed neutrons, Equation (1.3.11) would be a 7 x 7 determinant and Equation (1.3.14) would be enlarged to

$$\rho_o = \Lambda\, \omega + \sum_{i=1}^{6} \frac{\beta_i \omega}{\omega + \lambda_i} \tag{1.3.15}$$

The $\underline{v}$ of Equation (1.3.10) is the eigenvector associated with the eigenvalue ω. Now if we perform a similarity transformation $\underline{\underline{B}}$ on $\underline{\underline{A}}$, then both $\underline{\underline{A}}$ and $\underline{\underline{B}}^{-1}\ \underline{\underline{A}}\ \underline{\underline{B}}$ have the same characteristic equation. Also the fact the fact that the trace of $\underline{\underline{A}}$ is the sum of the eigenvalues of the matrix A is a useful check on the actual calculation of the eigenvalues of A. From this last fact it is apparent that the sum of the two roots ω_1 and ω_2 (the two eigenvalues) of Equation (1.3.13) is given by the relation

$$\omega_1 + \omega_2 = \frac{\rho_o - \beta}{\Lambda} - \lambda . \tag{1.3.16}$$

The eigenvectors $\underline{v}_1$ and $\underline{v}_2$ associated with ω_1 and ω_2 respectively are obtained by taking the cofactors of the element in any <u>row</u> of Equation (1.3.12). To see how this works, we set

$$\underline{v}_1 = \begin{bmatrix} v_{11} \\ \\ v_{21} \end{bmatrix} \text{ and } \underline{v}_2 = \begin{bmatrix} v_{12} \\ \\ v_{22} \end{bmatrix} ,$$

and have v_{11}, the cofactor of the element $a_{11} - \omega_1$, as

$$v_{11} = -(\lambda + \omega_1),$$

and the cofactor of the element λ of the first row as

$$v_{21} = -\beta/\Lambda .$$

Similarly, for the vector $\underline{v}_2$, we have for the second row of $|\underline{\underline{A}} - \underline{\underline{I}}\omega|$,

$$v_{12} = -\lambda , \qquad v_{22} = \frac{\rho_o - \beta}{\Lambda} - \omega_2 .$$

Notice that to get the $\underline{v}_1$, we use the eigenvalue ω_1 and for $\underline{v}_2$, we use the eigenvalue ω_2 in Equation (1.3.12). Therefore, the eigenvectors of $\underline{\underline{A}}$ are (to within a constant)

$$\underline{v}_1 = \begin{bmatrix} \lambda + \omega_1 \\ \\ \frac{\beta}{\Lambda} \end{bmatrix} \quad \text{and } \underline{v}_2 = \begin{bmatrix} \lambda \\ \\ \frac{\beta - \rho_o}{\Lambda} + \omega_2 \end{bmatrix} .$$

These vectors $\underline{v}_1$ and $\underline{v}_2$ are linearly independent since the eigenvalues are distinct.

Now since we have

$$\underline{\underline{A}}\ \underline{v}_i = \omega_i\ \underline{v}_i , \quad \text{for} \quad i = 1, 2 ,$$

it is apparent that the similarity transformation

$$\underline{\underline{A}}\ \underline{\underline{B}} = \underline{\underline{B}}\ \underline{\underline{D}} \tag{1.3.17}$$

holds, where D is a diagonal matrix having the eigenvalues as its elements,

i.e.

$$\underline{\underline{D}} = \begin{bmatrix} \omega_1 & & o \\ & \ddots & \\ o & & \omega_2 \end{bmatrix}$$

and

$$\underline{\underline{B}} = [\ \underline{v}_1 \ . \ \underline{v}_2 \ ] \ .$$

These two matrices are now completely determined for the one delayed neutron model.

If we make a transformation

$$\underline{\psi}(t) = \underline{\underline{B}}\ \underline{Z}(t), \qquad (1.3.18)$$

then

$$\frac{d\underline{Z}(t)}{dt} = \underline{\underline{B}}^{-1}\ \underline{\underline{A}}\ \underline{\underline{B}}\ \underline{Z}(t)$$

$$= \underline{\underline{D}}\ \underline{Z}(t)$$

and its solution is obviously

$$\underline{Z}(t) = \begin{bmatrix} e^{\omega_1 t} & o \\ & \\ o & e^{\omega_2 t} \end{bmatrix} \underline{Z}(o) = e^{\underline{\underline{D}}t}\ .\ \underline{Z}(o) \ ; \qquad (1.3.19)$$

Now using Equations (1.3.18) and (1.3.19) we have

$$\underline{\psi}(t) = \underline{\underline{B}}\ e^{\underline{\underline{D}}t}\ \underline{\underline{B}}^{-1}\ \underline{\psi}(o) \ . \qquad (1.3.20)$$

This is the solution of the kinetics equations. Notice now that the exponential $e^{\underline{\underline{D}}t}$ is a diagonal matrix so the separation of the equations is effected.

We now write out the details of this procedure. The transformation matrix $\underline{\underline{B}}$ is

$$\underline{\underline{B}} = [\underline{v}_1 \quad \underline{v}_2] = \begin{bmatrix} \lambda + \omega & \lambda \\ \beta/\Lambda & \frac{\beta-\rho_o}{\Lambda} + \omega_2 \end{bmatrix}$$

and the determinant of $\underline{\underline{B}}$ is

$$\det \underline{\underline{B}} = \omega_1\omega_2 + \frac{\beta-\rho_o}{\Lambda} \quad \omega_1 + \lambda\omega_2 - \frac{\lambda\rho_o}{\Lambda} . \tag{1.3.21}$$

The inverse of $\underline{\underline{B}}$ is

$$\underline{\underline{B}}^{-1} = \frac{1}{\det \underline{\underline{B}}} \begin{bmatrix} \frac{\beta-\rho_o}{\Lambda} + \omega_2 & -\lambda \\ -\frac{\beta}{\Lambda} & \lambda + \omega_1 \end{bmatrix}$$

The solution of the one delayed neutron group equations is obtained from Equations (1.3.20) and (1.3.8)

$$\underline{\psi}(t) = \frac{n(o)}{\det \underline{\underline{B}}} \begin{bmatrix} \lambda + \omega_1 & \lambda \\ \frac{\beta}{\Lambda} & \frac{\beta-\rho_o}{\Lambda} + \omega_2 \end{bmatrix}$$

$$\times \begin{bmatrix} e^{\omega_1 t} & o \\ o & e^{\omega_2 t} \end{bmatrix}$$

$$\times \begin{bmatrix} \frac{\beta-\rho_o}{\Lambda} + \omega_2 & -\lambda \\ -\frac{\beta}{\Lambda} & \lambda + \omega_1 \end{bmatrix}$$

$$\times \begin{bmatrix} 1 \\ \frac{\beta}{\Lambda\lambda} \end{bmatrix}$$

$$= \frac{n(t)}{\text{Det } \underline{\underline{B}}} \begin{bmatrix} (\omega_2 - \frac{\rho_o}{\Lambda})(\lambda + \omega_1)\, e^{\omega_1 t} + \frac{\omega_1 \beta}{\Lambda}\, e^{\omega_2 t} \\ \frac{\beta}{\Lambda}\,(\omega_2 - \frac{\rho_c}{\Lambda})\, e^{\omega_1 t} + (\frac{\beta-\rho_o}{\Lambda} + \omega_2)\, \frac{\omega_1 \beta}{\lambda\Lambda}\, e^{\omega_2 t} \end{bmatrix} \qquad (1.3.23)$$

Even though Equations (1.3.23) represent the exact solution of the problem, we are generally not concerned with $c(t)$ so we look only at the $n(t)$ equation. Also from Equation (1.3.13) we have

$$\omega_{1,2} = \frac{-(\beta-\rho_o + \Lambda\lambda) \pm \sqrt{(\beta-\rho_o + \Lambda\lambda)^2 + 4\,\lambda\Lambda\,\rho_o}}{2\,\Lambda} \qquad (1.3.24)$$

Note too that

$$\omega_1 \omega_2 = - \frac{\lambda \rho_o}{\Lambda} \tag{1.3.25}$$

and in agreement with Equation (1.3.16),

$$\omega_1 + \omega_2 = - \frac{\beta - \rho_o + \lambda \Lambda}{\Lambda} . \tag{1.3.26}$$

Now if we approximate ω_1 (the positive sign) and ω_2 (the negative sign) in Equation (1.3.24) by assuming that

$$(\beta - \rho_o + \lambda \Lambda)^2 >> 4 \lambda \rho_o \Lambda ,$$

which is true for most reactors, we have the radical of Equation (1.3.24) equal to

$$(\beta - \rho_o + \Lambda \lambda) \sqrt{1 + \frac{4\lambda \rho_o \Lambda}{(\beta - \rho_o + \lambda\Lambda)^2}}$$

or

$$\left[1 + \frac{1}{2} \frac{4\lambda\Lambda\rho_o}{(\beta-\rho_o + \lambda\Lambda)^2} + \cdots\right] (\beta - \rho_o + \lambda\Lambda) .$$

When this is used we obtain

$$\omega_1 \approx \frac{\lambda\rho_o}{\beta-\rho_o} \quad \text{and} \quad \omega_2 \approx - \frac{\beta-\rho_o}{\Lambda} . \tag{1.3.27}$$

Note that if ρ_o is positive, then ω_1 is also positive.

The determinant of $\underset{\sim}{B}$ is involved in the solution of n(t) so with the above approximations as well as by using Equation (1.3.25) we have

$$\det \underline{\underline{B}} \doteq -\frac{\lambda\rho_o}{\Lambda} + \frac{\beta-\rho_o}{\Lambda}\cdot\frac{\lambda\rho_o}{\beta-\rho_o} - \frac{\lambda}{\Lambda}(\beta-\rho_o) - \frac{\lambda\rho_o}{\Lambda} = -\frac{\lambda\beta}{\Lambda}. \qquad (1.3.28)$$

So Equation (1.3.23) becomes, using Equation (1.3.28),

$$n(t) = n(o)\left[\frac{\beta}{\beta-\rho_o}\, e^{\frac{\lambda\rho_o}{\beta-\rho_o}t} - \frac{\rho_o}{\beta-\rho_o}\, e^{-\frac{\beta-\rho_o}{\Lambda}t}\right] \qquad (1.3.29)$$

This approximate solution is very useful in obtaining checks on numerical solutions. The character of the solution is also exponential. This will play a role in our numerical technique.

Problem 1.3.1

Assume that $\rho(t) = 0$ in Equations (1.3.3) and (1.3.4) and obtain a solution of the kinetics equations.

Problem 1.3.2

Prove the theorems

a) $|\underline{\underline{B}}^{-1}\,\underline{\underline{A}}\,\underline{\underline{B}} - \omega\,\underline{\underline{I}}| = |\underline{\underline{A}} - \omega\underline{\underline{I}}|$

and

b) $\text{trace } \underline{\underline{A}} \doteq \sum_{i=1}^{2} \omega_i$

Problem 1.3.3

Why does Equation (1.3.17) follow from the eigenvector equation $\underline{\underline{A}}\,\underline{v} = \omega\,\underline{v}$?

Problem 1.3.4

Using the same approximations as used in developing Equation (1.3.29) establish a relation for $c(t)$.

1.4 Numerical Solution of the Kinetics Equations

The reactor kinetics equations are difficult to solve numerically by "standard" Runge-Kutta of predictor-corrector methods. The basic reason becomes apparent by looking at the one-delayed neutron group equations which we repeat as

$$\frac{dn(t)}{dt} = \frac{\rho(t)-\beta}{\Lambda} n(t) + \lambda c(t) \tag{1.4.1}$$

and

$$\frac{dc(t)}{dt} = \frac{\beta}{\Lambda} n(t) - \lambda c(t) . \tag{1.4.2}$$

The very short time response Λ of the prompt neutrons is of the order of 10^{-4} sec. whereas the delayed neutron time response is $\frac{1}{\lambda}$ or about 10 sec, a factor of 10^5 greater. The implication of these facts is that in order to obtain the prompt response, very small time steps, of the order of 10^{-4}sec, are required. But then before the delayed neutron term can come into play, many time steps are required. Also, to examine the response out to even one second, 10,000 steps of calculation would be required.

There are several methods that are used to ameliorate this difficult problem:

1) Using a Laplace transform technique.
2) Transferring the differential equations to integral equations.
3) Using the eigenvalue method.

We choose the last technique and refer to it as Hansen's method (4) after its originator. The method works for varying reactivity and can be extended

to systems with feedback.

The basic idea of Hansen's method is relatively simple. We again write Equations (1.4.1) and (1.4.2) as a matrix and set

$$\frac{d\psi(t)}{dt} = \underline{\underline{A}}\,\underline{\psi} \qquad (1.4.3)$$

where A and $\underline{\psi}$ are defined as in Section 3 of this module. We will also only perform the operations for our one delayed neutron group model. Now set

$$\underline{\underline{A}} = \underline{\underline{L}} + \underline{\underline{D}} + \underline{\underline{U}} \qquad (1.4.4)$$

where

$$\underline{\underline{L}} = \begin{bmatrix} 0 & 0 \\ \frac{\beta}{\Lambda} & 0 \end{bmatrix}, \quad \underline{\underline{D}} = \begin{bmatrix} \frac{\rho-\beta}{\Lambda} & 0 \\ 0 & -\lambda \end{bmatrix}$$

and

$$\underline{\underline{U}} = \begin{bmatrix} 0 & \lambda \\ 0 & 0 \end{bmatrix}$$

Of course, for all six delayed neutron groups these matrices would still have the same meaning. Equation (1.4.4) can be written using these definitions as

$$\frac{d\psi(t)}{dt} - \underline{\underline{D}}\,\underline{\psi}\,(t) = (\underline{\underline{L}} + \underline{\underline{U}})\,\underline{\psi}\,(t) \qquad (1.4.5)$$

Equation (1.4.5) does not appear any simpler to solve than Equation (1.4.3) and in fact it isn't. The reason for splitting it up in this fashion is to develop an iteration procedure. We assume we begin this calculation from a time t_0 and advance to a time t_1. We set ,

$$h = t_1 - t_0 \; . \qquad (1.4.6)$$

Since $\underline{\underline{D}}$ is a diagonal matrix, an integrating factor for Equation (1.4.5) is $e^{-\underline{\underline{D}}t}$ if the reactivity doesn't change much during the time interval h. Therefore, Equation (1.4.5) becomes

$$e^{-\underline{\underline{D}}t} \frac{d\underline{\psi}(t)}{dt} - e^{-\underline{\underline{D}}t} \underline{\underline{D}} \, \underline{\psi}(t) = e^{-\underline{\underline{D}}t} (\underline{\underline{L}} + \underline{\underline{U}}) \, \underline{\psi}(t) \; .$$

or

$$\frac{d}{dt} \left(e^{-\underline{\underline{D}}t} \underline{\psi}(t)\right) = e^{-\underline{\underline{D}}t} (\underline{\underline{L}} + \underline{\underline{U}}) \, \underline{\psi}(t) \qquad (1.4.6)$$

Integrating now from 0 to h we have

$$e^{-\underline{\underline{D}}t} \psi(t) \Big|_o^h = \int_o^h e^{-\underline{\underline{D}}t} (\underline{\underline{L}} + \underline{\underline{U}}) \, \underline{\psi}(t) \, dt$$

or

$$\psi(t_o + h) = e^{\underline{\underline{D}}h} \underline{\psi}(t_o) + \int_o^h e^{\underline{\underline{D}}(h-\Theta)} (\underline{\underline{L}} + \underline{\underline{U}}) \, \underline{\psi}(t_o + \Theta) d, \qquad (1.4.7)$$

where

$$t_o \leq \Theta \leq t_1 = t_o + h \; ,$$

and obviously in this interval

$$d\Theta = dt.$$

This is an integral equation since the function we're looking for is part of the integrand. In order to provide a reasonable approximation to $\underline{\psi}(t_o + \Theta)$, we recall that the analytical solution is exponential so we assume

$$\underline{\psi}(t_o + \Theta) = e^{\omega_o \Theta} \underline{\psi}(t_o) , \tag{1.4.8}$$

and ω_o is the largest eigenvalue of the matrix $\underline{\underline{A}}$. This means that we will have to solve the equation (notice this is simply the inhour equation)

$$|\underline{\underline{A}} - \omega \underline{\underline{I}}| = 0 \tag{1.4.9}$$

in each time interval for which the reactivity has changed since the reactivity will generally be a time dependent quantity.

Inserting Equation (1.4.8) into (1.4.7) we have

$$\underline{\psi}(t_o + h) = e^{\underline{\underline{D}}h} \underline{\psi}(t_o) + \int_o^h e^{\underline{\underline{D}}(h-\Theta)} (\underline{\underline{L}} + \underline{\underline{U}}) e^{\omega_o \Theta} \underline{\psi}(t_o) d\Theta$$

$$= e^{\underline{\underline{D}}h} \underline{\psi}(t_o) + \left[\int_o^h e^{\underline{\underline{D}}(h-\Theta)} e^{\omega_o \underline{\underline{I}} \Theta} d\Theta \right] (\underline{\underline{L}} + \underline{\underline{U}}) \underline{\psi}(t_o)$$

$$= e^{\underline{\underline{D}}h} \underline{\psi}(t_o) + \left[e^{\underline{\underline{D}}h} \cdot \int_o^h e^{(\omega_o \underline{\underline{I}} - \underline{\underline{D}})\Theta} d\Theta \right] (\underline{\underline{L}} + \underline{\underline{U}}) \underline{\psi}(t_o)$$

$$= e^{\underline{\underline{D}}} \underline{\psi}(t_o) + (\omega_o \underline{\underline{I}} - \underline{\underline{D}})^{-1} \left[e^{\omega_o h \underline{\underline{I}}} - e^{\underline{\underline{D}}h} \right] (\underline{\underline{L}} + \underline{\underline{U}}) \underline{\psi}(t_o) . \tag{1.4.10}$$

If we write

$$\psi(t_o) = \psi_j \tag{1.4.11}$$

and

$$\psi(t_o + h) = \psi_{j+1}$$

Then Equation (1.4.10) becomes

$$\underline{\psi}_{j+1} = e^{\underline{\underline{D}}h} \underline{\psi}_j + (\omega_o \underline{\underline{I}} - \underline{\underline{D}})^{-1} \left[e^{\omega_o h \underline{\underline{I}}} - e^{\underline{\underline{D}}h} \right] (\underline{\underline{L}} + \underline{\underline{U}}) \underline{\psi}_j \tag{1.4.11}$$

$$\equiv \underline{\underline{G}}\, \underline{\psi}_j . \tag{1.4.12}$$

This $\underline{\underline{G}}$ matrix obviously represents

$$\underline{\underline{G}} = e^{\underline{\underline{D}}h} + (\omega_o \underline{\underline{I}} - \underline{\underline{D}})^{-1} \left[e^{\omega_o \underline{\underline{I}} h} - e^{\underline{\underline{D}}h} \right] (\underline{\underline{L}} + \underline{\underline{U}}), \tag{1.4.13}$$

and it can be written as

$$\underline{\underline{G}} = \begin{bmatrix} e^{\frac{\rho-\beta}{\Lambda} h} & \dfrac{e^{\omega_o h} - e^{+\frac{\rho-\beta}{\Lambda} h}}{\omega_o - (\frac{\rho-\beta}{\Lambda})} \lambda \\ \dfrac{e^{\omega_o h} - e^{-\lambda h}}{\omega_o + \lambda_1} \dfrac{\beta}{\Lambda} & e^{-\lambda h} \end{bmatrix} \tag{1.4.14}$$

For completeness, if there are N delayed neutron groups, we include the following relation for $\underline{\underline{G}}$,

$$\underline{\underline{G}} = \begin{bmatrix} e^{\frac{\rho-\beta}{\Lambda}h} & \dfrac{e^{\omega_o h} - e^{\frac{\rho-\beta}{\Lambda}h}}{\omega_o - (\frac{\rho-\beta}{\Lambda})}\lambda_1 & \cdots & \dfrac{e^{\omega_o h} - e^{\frac{\rho-\beta}{\Lambda}h}}{\omega_o - (\frac{\rho-\beta}{\Lambda})}\lambda_N \\ \dfrac{e^{\omega_o h} - e^{-\lambda_1 h}}{\omega_o + \lambda_1}\dfrac{\beta_1}{\Lambda} & e^{-\lambda_1 h} & \cdots & 0 \\ \vdots & & & \vdots \\ \dfrac{e^{\omega_o h} - e^{-\lambda_N h}}{\omega_o + \lambda_N}\dfrac{\beta_N}{\Lambda} & \cdots & \cdots & e^{-\lambda_N h} \end{bmatrix} \tag{1.4.15}$$

If there are six delayed neutron groups, it is apparent that $\underline{\underline{G}}$ will be a 7 x 7 matrix. Also this iteration technique as given by Equation (1.4.12) is unconditionally stable and yields the asymptotically correct eigen-solution.

The numerical procedure for the solution of the kinetics equations is thus:

1. Determine the number of delayed neutron groups desired and read in the pertinent parameters such as Λ_1, β_1 etc. Also choose a time step "h".
2. Construct the vector $\underline{\psi}(0)$. This will usually be

$$\underline{\psi}(0) = n(0)\begin{bmatrix} 1 \\ \beta_1/\lambda_1\Lambda \\ \vdots \\ \beta_N/\lambda_N\Lambda \end{bmatrix}$$

3. Determine the largest eigenvalue of the equation

$$|\underline{\underline{A}} - \omega\underline{\underline{I}}| = 0.$$

This is a rather difficult step since it means solving an algebraic equation of perhaps degree 7 to determine its largest root ω_o.

4. Construct the $\underline{\underline{G}}$ matrix using Equation (1.4.15).

5. Determine the vector $\underline{\psi}_1$ where

$$\underline{\psi}_1(h) = G\,\underline{\psi}(0).$$

6. Repeat the above steps starting with step 3.

The technique is not involved and can yield very accurate results.

The determination of the root of an algebraic equation needs some discussion. We choose the Newton-Raphson technique to solve the equation

$$f(\omega) = \sum_{n=0}^{N} a_n \omega^n = 0 \quad \text{for } 1 \leq N \leq 7. \qquad (1.4.16)$$

Let the N roots of Equation (1.4.17) be labelled $\omega_o, \omega_1, \ldots, \omega_N$ where the roots are ordered such that $\omega_o > \omega_1 > \omega_2 \ldots > \omega_7$. For our problem, all the roots are real and there will only be one positive root depending on whether ρ is positive at the time step of interest. We are interested only in ω_o. Also we assume that the interval of interest in the roots is limited by

$$\omega_o \leq \left|\frac{\rho_o}{A}\right|$$

and

$$|\omega_N| \leq |\lambda_N|,$$

with λ_N the decay constant corresponding to the shortest lived delayed neutron group. The reactivity can range from negative β to positve β in

most practical situations. Note that if

$$\rho = 0 \text{ then } \omega_o = 0$$

and if $\rho = \beta$, then $\omega_o \to \infty$ while if $\rho = -\beta$, then $\omega_o \to -\lambda_1$. For the last situation, regardless of the amount of negative reactivity introduced into the reactor, the reactor cannot shut down faster than a period of

$$T = \omega_o^{-1} = \frac{1}{\lambda_1} \cong 80 \text{ sec.}$$

The Newton-Raphson method is relatively simple to use. Assume that we can expand $f(\omega)$ in a Taylor series about ω_o, where ω_o is the root of interest. Then

$$f(\omega_o) = f(\omega_{o_1}) + h \frac{df(\omega_{o_1})}{d\omega} + \frac{h^2}{2} \frac{d^2 f(\omega_{o_1})}{d\omega^2} + \ldots \qquad (1.4.17)$$

The ω_{o_1} is a first "guess" at the solution which we assume to be $\rho(t)/\Lambda$. If we truncate Equation (1.4.17) after the first two terms on the right, we have

$$f(\omega_o) \cong f(\omega_{o_1}) + h \frac{df}{d\omega}(\omega_{o_1}) = 0$$

or

$$h = \frac{-f(\omega_{o_1})}{\frac{df}{d\omega}(\omega_{o_1})} \qquad (1.4.18)$$

The next approximation to ω_o is then

$$\omega_{o_2} = \omega_{o_1} + h \qquad (1.4.19)$$

and then Equation (1.4.18) is used again. If this iteration procedure is used a sufficient number of times, then the roots of Equation (1.4.16) can be obtained.

Problem 1.4.1

Use the Newton-Raphson method to solve the equation

$$x^2 + 3x - 8 = 0.$$

Check by the quadratic formula.

Problem 1.4.2

Extend the Newton-Raphson method to systems of equations. In particular if

$$f_1\ (x_1, x_2) = 0$$

and

$$f_2\ (x_1, x_2) = 0$$

then show that if h_1 and h_2 are the increments to the assumed roots,

$$h_1 = \frac{f_2 \dfrac{\partial f_1}{\partial x_2} - f_1 \dfrac{\partial f_2}{\partial x_2}}{\det \underline{\underline{J}}}$$

and

$$h_2 = \frac{f_1 \dfrac{\partial f_2}{\partial x_2} - f_2 \dfrac{\partial f_1}{\partial x_1}}{\det \underline{\underline{J}}}$$

where $\underline{\underline{J}}$ is the Jacobian matrix, i.e.

$$\underline{\underline{J}} = \begin{bmatrix} \dfrac{\partial f_1}{\partial x_1} & \dfrac{\partial f_1}{\partial x_2} \\ \\ \dfrac{\partial f_2}{\partial x_1} & \dfrac{\partial f_2}{\partial x_2} \end{bmatrix}$$

and det $\underline{\underline{J}}$ is the determinant of the Jacobian matrix.

Using this formulation find the solution to the equations

$$f_1(x_1, x_2) = \sin x_1 x_2 - x_1 + x_2^2 = 0$$

$$f (x_1, x_2) = 2x_1^2 - x_2^2 + x_1 x_2 = 0.$$

1.5 The Computer Program and the Kinetics Equations

As has already been indicated, the kinetics equations offer a challenge for their successful computer solution. The purpose of this section is to indicate what models the program will solve. The program will solve the kinetics equations by means of Hansen's method for the following reactivity inputs:

1) A constant reactivity,

$$\rho(t) = \rho_o. \quad (1.5.1)$$

2) Sinsusoidal variation of reactivity,

$$\rho(t) = \rho_o \sin b_2 t, \quad (1.5.2)$$

where ρ_o and b_2 must be input.

3) Linear reactivity insertion to approximate the insertion or withdrawal of a control rod:

$$\rho(t) = \rho_o(1 + b_3 t). \quad (1.5.3)$$

These inputs are ~~the~~ most common and can be used to simulate many situations. In each of the above situations ρ_o must be read into the computer as well as the "b" or rate of insertion in sec^{-1}. The reactivity units are in dollars. A dollar of reactivity is the amount inserted or withdrawn which equals the delayed neutron fraction. For example, a reactivity of 50¢ for U-235 where $\beta \doteq 0.0065$ is 0.00325 whereas for Pu-239, it is 0.00135 since $\beta = 0.0027$.

The program also has an option of either doing the calculations for

U-235 or Pu-239. The delayed neutron decay constants are taken from Keepins' data and are given in Table 1.5.1.

Table 1.5.1 Delayed-Neutron Half-Lives, Decay Constants and Yields from Thermal Fission of U-235 and Pu-239.

	URANIUM-235			PLUTONIUM-239		
Group Index	Half-Life (sec)	Decay Constant λ (sec^{-1})	Relative Abundance $\beta i/\beta$	Half-Life (sec)	Decay Constant λ (sec^{-1})	Relative Abundance $\beta i/\beta$
1	55.72	0.0124	0.033	54.28	0.0128	0.035
2	22.72	0.0305	0.219	23.04	0.0301	0.298
3	6.22	0.111	0.196	5.60	0.124	0.211
4	2.30	0.301	0.395	2.13	0.325	0.326
5	0.610	1.14	0.115	0.618	1.12	0.086
6	0.230	3.01	0.042	0.257	2.69	0.044
		β=0.0065			β=0.0027	

The computer program for this module is good only for low power reactors since feedback effects are not taken into account. It does give some idea of the behavior of the power of the reactor for various reactivities as well as for the two different fuels.

I.6 Input-Output Data for Code FUMOKI

The input data required for the program are presented below:

Card 1. IFUEL indicates the type of fuel the reactor has. It can either be ^{235}U or ^{239}Pu. If IFUEL = 5 - its Uranium -235

IFUEL = 9 - the fuel is Plutonium 239.

The format is I2.

Card 2. NN - The number of groups of delayed neutrons desired. NN can be any integer from 1 to 6.

The format is again I2.

Card 3. NRO - Type of reactivity inserted into the reactor.

The format is I2.

If NRO = 1; $\rho(t) = \rho_0$,

NRO = 2, $\rho(t) = \rho_0 \sin b_2 t$,

NRO = 3; $\rho(t) = \rho_0(1 + b_3 t)$.

Card 3 must have the reactivity ρ_0, in dollars, read in with an F 12.9 format. The b_2 and b_3 are read in with units of sec^{-1}. The b's are read with an F 12.9 format on this card.

Card 4. XL - Neutron generation time.

XL must be in seconds and is in an F 12.9 format.

Card 5. TIME: - the total time period for which the solution is desired (seconds)

H - The time step desired, again in seconds. This cannot be taken arbitrarily large. Generally one should choose an H of between 0.005 and 0.05 sec. Both of these are F 10.5 formats.

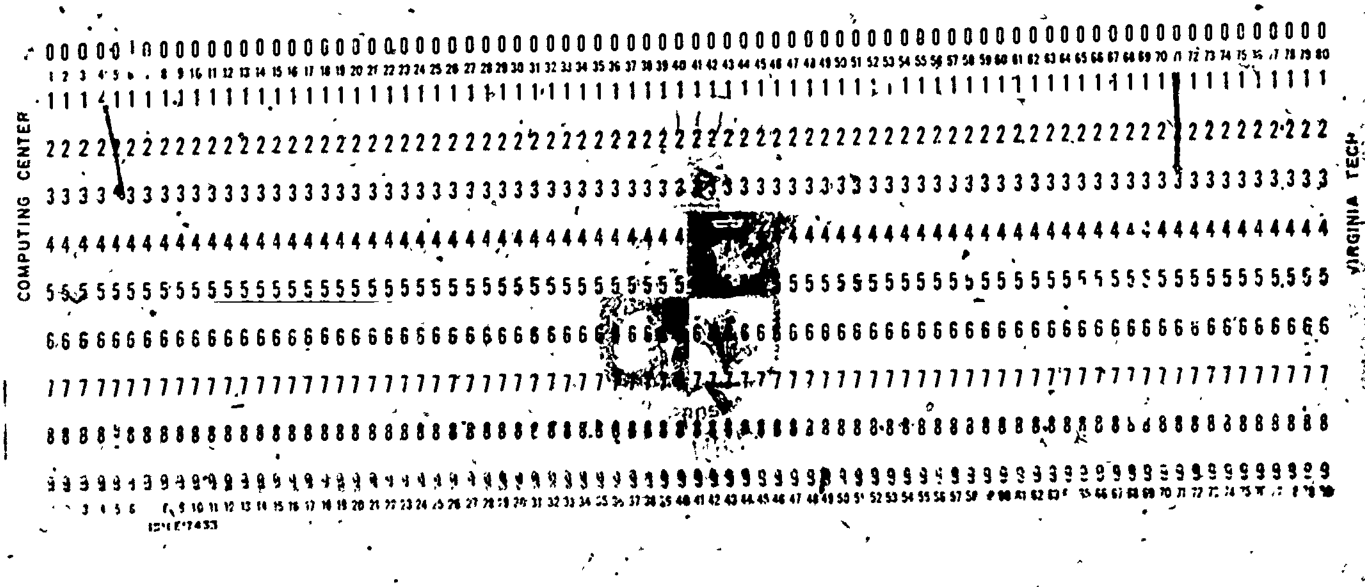
Card 6. IG - Output indicator. This indicates whether the user would like to print out the G matrix, the flux and time step or not. If IG = 1, then the flux, "G" matrix and time step are all printed. If IG = 2, G is not printed out. This is an I2 format.

Card 7. XN - The initial neutron density.

XN can be anything you choose but it is in an F 10.5 format.

An example of an input data set is given below. This is an example where we use Uranium fuel, with one group of delayed neutrons for a constant reactivity input of about 34¢.

1
2
3
4
5
6
7



The core requirements for FUMOKI are about 40 K bytes and the compilation time increases approximately with the square of the number of delayed neutron groups, all other parameters being the same.

The output of the program is simple. The time, neutron density and G matrix can be read out. Also a plot of the density versus time is printed out.

Problem 1.6.1

Run the sample problem, i.e. the data shown.

Problem 1.6.2

Find the neutron density if you start with 100 neutrons/cm^3 for a ^{239}Pu reactor with all six delayed neutron groups. Obtain the solution for a 5 second time span and do the same problem with H = 0.01 and 0.1 sec. Compare the results. Do not write the $\underline{\underline{G}}$ matrix.

Problem 1.6.3

Run FUMOKI for the case for three delayed neutron groups for ^{239}Pu, with a neutron generation time of 10^{-4} sec and the reactivity

$$\rho(t) = \$0.25 \sin 5t,$$

Run it for a total time of 1 second, time intervals of 0.01 sec and an initial relative flux of 1.000. Notice that the reactivity at 0.5 sec is \$0.17 and the power has risen to 1.275.

If you do the exact same problem as above for 2 delayed neutron groups notice that the power has become 1.268 at 0.5 sec.

Problem 1.6.4

Given a reactor fueled with ^{239}Pu and assume that you wish to use a 5-delayed-neutron-group model. For a relative power of 1.000 initially, show that after one second the power is 2.157 if a rod is being removed such that the reactivity inserted is

$$\rho(t) = \rho_o(1 + 5t).$$

All other parameters are as in problem 1.6.3.

Problem 1.6.5

Assume a ^{235}U fueled system with a reactivity variation of

$$\rho(t) = \$0.25 \sin 5t.$$

If the reactor is to be simulated for three seconds after the reactivity is inserted and if all parameters are the same as in example 1.6.3 except for the number of delayed neutron groups then:

a) use the two delayed neutron model to obtain the power as a function of time

and

b) use the five delayed neutron model to obtain the power variation with time.

How much longer did the computer take to do case b) than case a)?

Answer: About 10 times as long. On the IBM 370, the times were 11.03 sec and .97 sec respectively. Notice also that the power at the end of 3.0 sec is 0.8154 and 0.8161 for the 2 and 5 group cases respectively.

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List of Symbols for FUMOKI

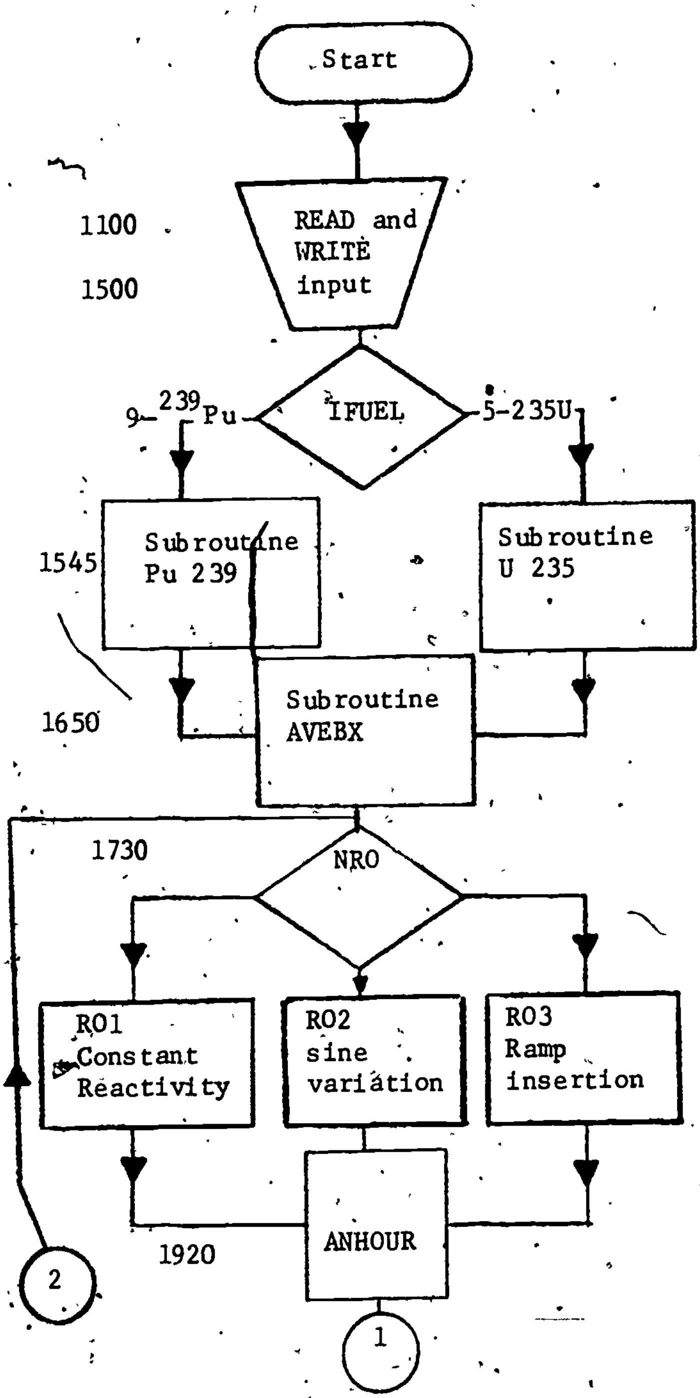
The following symbols are listed in the order of their appearance in program FUMOKI.

IFUEL		Type of fuel (^{235}U or ^{239}Pu) (5 or 9)
NN		Number of delayed neutron groups
NRO	$\rho(t) = \rho_o$ $\rho_o \sin b_2 t$ $\rho_o(1 + b_3 t)$	Type of reactivity to be inserted. 1 = constant reactivity 2 = sine insertion 3 = ramp insertion
RO	ρ_o	Reactivity inserted
B1,B2,B3	b_1, b_2, b_3	Rate or period at which reactivity is inserted.
X(I)	λ	precursor decay constants
XL	Λ	neutron generation time
TIME		total time the reactor transient (s) is to be simulated
H	Δt	the time step
IG		output option to print the $\underline{\underline{G}}$ matrix after 1st iteration(to print any other iteration change statement MKI 2520)
XN	n_o	initial relative power
B(I)		fraction of neutrons for each group
BB		total delayed neutron fraction
TH		cumulative time used in TPLOT
LL		total number of time steps H fits into TIME
R	$\rho(t)$	Reactivity at any time
MM		number of coefficients in the inhour equation. (number of delayed neutron groups + 2)

AW(I)		coefficients of the inhour polynomial
M		degree of the inhour polynomial (number of delayed neutron groups + 1)
WO	ω_o	largest eigenvalue of $\left\|\underline{\underline{A}} - \omega\underline{\underline{I}}\right\| = 0$
XNCO(I), GNXO(I)	$\underline{\phi}$	column vector $\underline{\phi}$
XXNC2		logarithm of power
GG,MAD		power
MOD(I)	H*(step number)	dimensioned TH used in TPLOT
Z1(I)		precursor density of delayed neutron group 1
Z2(I)		reactivity
AVEBX		subroutine to obtain the average β and λ depending on how many delayed neutron groups are desired
ANHOUR		subroutine to calculate the coefficients of the inhour equation
GMTRX		a subroutine which forms the $\underline{\underline{G}}$ matrix
GXN		subroutine which multiplies $\underline{\underline{G}}$ by column vector $\underline{\phi}$, i.e. $G\underline{\phi}_o$
POLRT		A Newton-Raphson iteration technique is used to obtain the roots of the inhour equation to obtain ω_o
TPLOT		A subroutine to plot the vector $\underline{\phi}$ up to and including five dependent variables. The reactivity, logarithm of the power, power and some of the delayed neutron precursors are plotted.

Flow Chart for FUMOKI

Statement Numbers



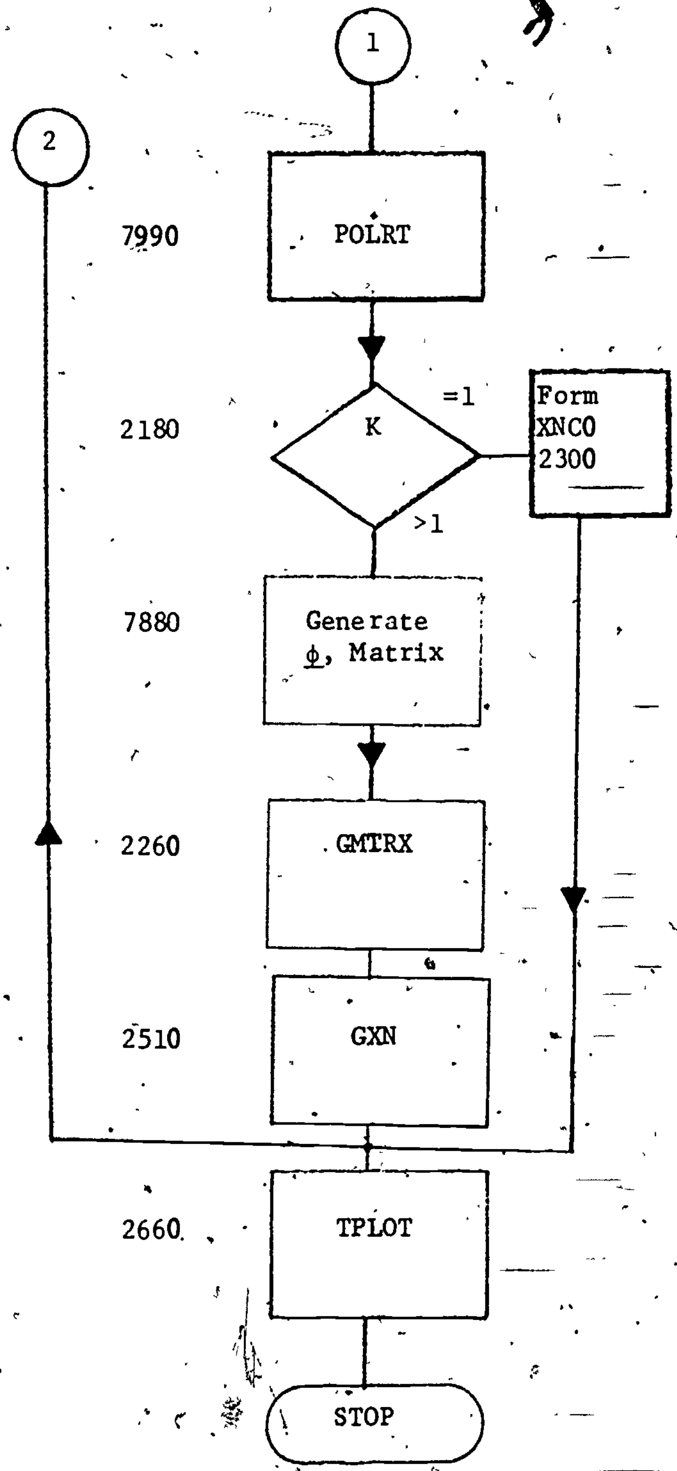
Obtains six group β's and λ's for isotope.

Calculates average β's and λ's depending on the number of delayed groups desired.

The type of reactivity variation desired determines which path to take.

Routines which determine the reactivity as functions of time.

Subroutine which calculates the polynomial form of the inhour equation.



7990 Subroutine to determine the largest root of the inhour equation using Newton-Raphson method.

2180 Subroutine which forms initial $\underline{\phi}$ matrix; $\underline{\phi}_o$

7880 Generates the $\underline{\phi}$ matrix for all cases except the first

2260 Forms the $\underline{\underline{G}}$ matrix

2510 Forms the product $\underline{\underline{G}}\ \underline{\phi}$

2660 Plots the power, etc.

```
//WATFIV SMART,OMEGA,PAGES=80,TIME=150
C       MAIN PROGRAM
C
C
C                         KINETICS MODULE,
C
C
C     CODE NAME      F U M O K I  .
C     OBJECTIVE    TO OBTAIN THE KINETICS EQUATION WITHOUT FEEDBACK AND
C               SOLVE THE KINETICS EQUATIONS NUMERICALLY FOR ONE , TO
C               SIX DELAYED NEUTRON GROUPS FOR TIME VARYING REACTIVITY
C               INSERTIONS .
C               THE KINETIC EQUATION DERIVED FROM ONE GROUP DIFFUSION
C               EQUATIONS .
C               FEEDBACK EFFECTS AND SPATIAL DEPENDENT EFFECTS ARE NOT
C               INCLUDED .
C
C     ASSUMPTION  THE BASIC ASSUMPTION OF THE METHOD IS THAT NEUTRON AND
C               PRECURSOR DENSITIES BEHAVE EXPONENTIALLY WITH A FREQUENCY
C               CHARACTERISTICS OF THE ASYMPTOTIC FREQUENCY CORRESPON-
C               DING TO THE REACTIVITY .
C
C     PROGRAM      WRITTEN IN SINGLE PRECISION
C
C               GENERAL DESCRIPTION OF PARAMETERS
C
C     SYMBOL  IN/OUT/V    DECRIPTION                 REAL/INT.   UNIT
C
C     IFUEL     IN         TYPE OF FUEL                  I         -
C                       5 = U-235
C                       9 = PU-239
C     NN        IN         GROUP DELAYED NEUTRON         I         -
C                       FROM 1 TO 6 .
C     NRO       IN         TYPE OF REACTIVITY            I         -
C                       1= CONSTANT
C                       2= RSIN(B2*T)
C                       3= R(1+B3*T)
C     B1        IN         CONSTANT B2 OR B3             R       1/SEC
C                       IN  NRO
C     B         OUT        FRACTION OF DELAYED           R         -
C                       NEUTRON
C     X         OUT        PRECURSOR DECAY               R       1/SEC
C                       CONSTANT
C     XL        IN         NEUTRON GENERATION            R        SEC
C                       TIME
C     R         V          REACTIVITY                    R         -
C     A         V          COEFFICIENT OF INHOUR         R         -
C                       FORMULA IN POLYNOMIAL
C                       FORM
C     WO        V          RECIPROCAL OF PERIOD          R       1/SEC
C     T         V          STABLE PERIOD                 R        SEC
C     G         V          G - MATRIX                    R         -
C     XN        V          NEUTRON DENSITY               R      N/CM**3
C     C         V          PRECURSOR DENSITY             R      N/CM**3
C     TIME      IN         TOTAL TIME                    R        SEC
C     H         IN         TIME INCREMENT                R        SEC
C
C
C     IN        - INPUT
C     OUT       - OUTPUT
```

```
C     V       - VARIABLES
C
C
C     --------------THIS PROGRAM  IS LIMITED TO DELAYED NEUTRONS FROM TH
C                   THERMAL FISSION OF U-235 AND PU-239 FOR ANY NUMBER OF
C                   GROUPS FROM ONE TO SIX .
C
C                   SUPPORTING ROUTINE    U235,PU239,AVEBX
C                                         RO1,RO2, AND RO3 .
C
C
C
1     REAL MAD,MOD
2     DIMENSION Z1(100), Z2(100), GG(100), MAD(100), MOD(100), XXNC2(100
     1), HH(100) .
3     DIMENSION B(6), XX(6), BX(6), X(6), YR(6), YX(6), A(8), OER1(8), F
     1W1(8), G(4,4), AW(8), ROOT1(7), COF(7), W(7), XNCO(7), GNXCO(7), C
     2(6)
C
C     ---------------DEFINE REACTIVITY AS A FUNCTION OF TIME .
C
4     RO2(RO,T,B2)=RO*SIN(B2*T)
5     RO3(RO,T,B3)=RO*(1.0+B3*T)
C
C
C                        INPUT PARAMETER DATA
C
C
C      110        IFUEL   EITHER 5 OR 9 .
C                         5 - FUELED U-235
C                         9 - FUELED U-239
C      120        NN      GROUP DELAYED NEUTRON
C      130        NRO     TYPE OF REACTIVITY
C                         1 - CONSTANT REACTIVITY
C                         2 - REACTIVITY AS A FUNCTION OF TIME .
C                           R = RO*SIN(B2*T)
C                         3 - REACTIVITY AS A LINEAR FUNCTION OF T .
C                           R = RO*(1+B3*T)
C                 RO      CONSTANT / INITIAL REACTIVITY .
C                 B1      EITHER B2 OR B3 IN NRO
C      140        XL      NEUTRON GENERATION TIME
C      150        TIME    TOTAL TIME USED
C                 H       TIME INTERVAL
C      160        IG      OUTPUT SIGNAL FOR G-MATRIX
C                           1 - PRINT OUT G-MATRIX
C                           2 - DON'T .
C      170        XN      INITIAL RELATIVE FLUX
C
C      ****INPUT REACTIVITY IN DOLLAR UNIT *******
C
C
C     ---------------READ THE INPUT DATA, STEP 1 P. 26
6     READ (5,110) IFUEL
7     READ (5,120) NN
8     READ (5,130) NRO,RO,B1
9     READ (5,140) XL
10    READ (5,150) TIME,H
11    READ (5,160) IG
12    READ (5,170) XN
13    WRITE (6,20)
14    WRITE (6,30) IFUEL,NN
```

```
      WRITE (6,40) XL                                                  MK1 1210
      WRITE (6,50) NRE,PO,NPO,B1                                       MK1 1220
      WRITE (6,60) TIME,H                                              MK1 1230
      WRITE (6,70) IO,XN                                               MK1 1240
      WRITE (6,100)                                                    MK1 1250
   20 FORMAT (1H1,60H******************************** KINETICS MODULE 1 **MK1 1260
     1********************,///,20H********************,/,1X,16H INPUT PARAMK1 1270
     2METER,/,20H********************,/////)                           MK1 1280
   30 FORMAT (2X,18H TYPE OF FUEL U-23,I1,//,2X,29H NUMBER OF DELAYED NEMK1 1290
     1UTRON = ,I2,/)                                                   MK1 1300
   40 FORMAT (2X,27H NEUTRON GENERATION TIME = ,F11.5,2X,'(SEC)',/)    MK1 1310
   50 FORMAT (2X,22H TYPE OF REACTIVITY = ,I2,//,2X,10H RO =    $   ,F15.3,MK1 1320
     15X,'B',I1,' = ',F11.5,/)                                         MK1 1330
   60 FORMAT (2X,19H TOTAL TIME USED = ,F10.3,2X,'(SEC)',5X,'  ,  TIME IMK1 1340
     <NTERVAL = ',F7.3,'  (SEC)',/)                                    MK1 1350
   70 FORMAT (2X,17H OUTPUT OPTION = ,I2,//,2X,25H INITIAL RELATIVE FLUXMK1 1360
     1 = ,F5.3,//)                                                     MK1 1370
   80 FORMAT (5X,22H SIX GROUP LAMBDA ARE ,2X,6(F7.4,2X),//)           MK1 1380
   90 FORMAT (5X,20H SIX GROUP BETA ARE ,2X,6(F7.5,2X),//)             MK1 1390
  100 FORMAT (/////,39H********** END OF INPUT DATA **********,/////)  MK1 1400
  110 FORMAT (I2)                                                      MK1 1410
  120 FORMAT (I2)                                                      MK1 1420
  130 FORMAT (I10,2(F10.5))                                            MK1 1430
  140 FORMAT (F10.6)                                                   MK1 1440
  150 FORMAT (2(F10.5))                                                MK1 1450
  160 FORMAT (I2)                                                      MK1 1460
  170 FORMAT (F10.5)                                                   MK1 1470
C                                                                      MK1 1480
C     --------------COMPUTE # OF ITERATION AND TYPE OF FUEL USED       MK1 1490
C                                                                      MK1 1500
      NI=1                                                             MK1 1510
      BB=0.0                                                           MK1 1520
      TH=0.0                                                           MK1 1530
      H1=H                                                             MK1 1540
      LL=(TIME+H1/2.)/H1                                               MK1 1550
      IF (IFUEL-5) 190,180,190                                         MK1 1560
  180 CALL U235 (B,X)                                                  MK1 1570
      GO TO 200                                                        MK1 1580
  190 CALL PU239 (B,X)                                                 MK1 1590
  200 WRITE (6,80) (X(I),I=1,6)                                        MK1 1600
      WRITE (6,90) (B(I),I=1,6)                                        MK1 1610
C                                                                      MK1 1620
C     --------------COMPUTE THE AVERAGE BETA AND LAMDA FOR THE PARTICULAMK1 1630
C                   FUEL .                                             MK1 1640
C                                                                      MK1 1650
      CALL AVESX (B,X,6,NN)                                            MK1 1660
      WRITE (6,210)                                                    MK1 1670
  210 FORMAT (/,5X,27H THE AVERAGE BETA AND LAMDA,/,5X,28H-------------MK1 1680
     1---------------,/)                                               MK1 1690
      DO 220 L=1,NN                                                    MK1 1700
  220 WRITE (6,230) L,B(L),L,X(L)                                      MK1 1710
  230 FORMAT (6X,2HB(,I2,3H)= ,F11.5,5X,7HLAMBDA(,I2,4H) = ,F11.5,/)   MK1 1720
      DO 460 K=1,LL                                                    MK1 1730
      GO TO (240,250,260), NRE                                         MK1 1740
C                                                                      MK1 1750
C     --------------SEE EQUATION 1.5.1                                 MK1 1760
C                                                                      MK1 1770
  240 R=RO                                                             MK1 1780
      GO TO 270                                                        MK1 1790
C                                                                      MK1 1800
```

```
      C      ---------------SEE EQUATION 1.5.2                                  MK1 1810
      C                                                                         MK1 1820
 57   250    R=RO2(RO,TH,B1)                                                    MK1 1830
 58          GO TO 270                                                          MK1 1840
      C                                                                         MK1 1850
      C      ---------------SEE EQUATION 1.5.3                                  MK1 1860
      C                                                                         MK1 1870
 59   260    R=RO3(RO,TH,B1)                                                    MK1 1880
      C                                                                         MK1 1890
      C      ---------------COMPUTE THE COEFFICIENT OF INHOUR FORMULA, SEE EQUATMK1 1900
      C                ON 1.3.13 WHERE NN=1                                     MK1 1910
      C                                                                         MK1 1920
 60   270    CALL ANHOUR (XL,X,B,R,A,NN)                                        MK1 1930
 61          MM=NN+2                                                            MK1 1940
 62          IF (K.EQ.1) WRITE (6,280)                                          MK1 1950
 63   280    FORMAT (/,4X,32H COEFFICIENTS OF INHOUR FORMULA ,/,4X,35H---------MK1 1960
            1--------------------------,/)                                      MK1 1970
 64          DO 300 I=1,MM                                                      MK1 1980
 65          MW=MM-I+1                                                          MK1 1990
 66          AW(MW)=A(I)                                                        MK1 2000
 67          IF (K.GT.1) GO TO 300                                              MK1 2010
 68          WRITE (6,290) I,AW(MW)                                             MK1 2020
 69   290    FORMAT (/,5X,2HA(,I2,5H) =   ,F11.4)                              MK1 2030
 70   300    CONTINUE                                                           MK1 2040
 71          M=NN+1                                                             MK1 2050
      C                                                                         MK1 2060
      C      ---------------COMPUTE THE ROOT OF THE POLYNOMIAL WITH MM COEFFICIEMK1 2070
      C                AND DETERMINE THE LARGEST EIGEN VALUE WO IN STEP 3      MK1 2080
      C                                                                         MK1 2090
 72          CALL POLRT (A,COF,M,W,ROOTI,IER,MM)                                MK1 2100
 73          WO=W(1)                                                            MK1 2110
 74          DO 310 J=1,M                                                       MK1 2120
 75          VO=AMAX1(W(J),WO)                                                  MK1 2130
 76   310    WO=VO                                                              MK1 2140
 77   340    IF (K.NE.1) GO TO 370                                              MK1 2150
 78          WRITE (6,350) WO                                                   MK1 2160
 79   350    FORMAT (//,6X,' THE LARGEST EIGEN-VALUE  =  ',E11.4,//)            MK1 2170
      C                                                                         MK1 2180
      C      ---------------FORM THE INITIAL VECTOR COLUMN PHI .                MK1 2190
      C                INITIALIZE NEUTRON DENSITY, T=0 AND PRECURSOR DENSITY.   MK1 2200
      C                                                                         MK1 2210
 80          XNCO(1)=XN                                                         MK1 2220
 81          DO 360 I=2,M                                                       MK1 2230
 82          J1=I-1                                                             MK1 2240
 83          BB=BB+B(J1)                                                        MK1 2250
 84          C(J1)=B(J1)/(X(J1)*XL)                                             MK1 2260
 85   360    XNCO(I)=C(J1)*XN                                                   MK1 2270
 86   370    IF(K.GT.1)GG(K)=GNXCO(1)                                           MK1 2280
 87          GG(K)=XNCO(1)                                                      MK1 2290
 88          RR=R*BB                                                            MK1 2300
 89          IF(K-1)380,400,380                                                 MK1 2310
 90   380    H=H1                                                               MK1 2320
 91          DO 390 J1=1,M                                                      MK1 2330
 92   390    XNCO(J1)=GNXCO(J1)                                                 MK1 2340
 93          GO TO 410                                                          MK1 2350
 94   400    H=0.0                                                              MK1 2360
      C                                                                         MK1 2370
      C      ---------------CONSTRUCT THE G MATRIX CORRESPONDING TO EQUATION    MK1 2380
      C                1.4.15 AND STEP 4 AND 5 PAGE 26 AND MULTIPLY THE G-MA -  MK1 2390
      C                TRIX BY INITIAL VECTOR .                                 MK1 2400
```

```
C
  410 CALL GMTRX (RR,B,X,XL,WQ,M,H,G)
      IF (K.NE.2.AND.IG.NE.1) GO TO 430
      WRITE (6,480)
      DO 420 I=1,M
  420 WRITE (6,490) (G(I,II),II=1,M)
  430 CALL GXN (G,XNCO,M,GNXCO)
      IF (K.EQ.1) WRITE (6,440) NN
  440 FORMAT (///,2X,2HSP,3X,4HTIME,6X,1HR,7X,5HPOWER,5X,I2,1X,21HGROUP
     1DELAYED NEUTRON,/,1X,4H(   ),1X,5H(SEC),5X,3H($),5X,7H(  -  ),7X,10
     2H(RELATIVE),//)
      WRITE (6,450) K,TH,P,(GNXCO(I),I=1,M)
  450 FORMAT (1X,I3,2X,F7.3,2X,F5.2,7(2X,E11.4))
      TH=TH+H
      MOD(K)=TH
      DG=GG(K)
      MAD(K)=DG
      Z1(K)=GNXCO(2)
      Z2(K)=P
  460 XXNC2(K)=ALOG(DG)
      WRITE (6,500)
      CALL TPLOT (MOD,MAD,Z1,XXNC2,Z2,IL)
  480 FORMAT (/,6X,14H THE G-MATRIX ,/,6X,15H ------------- ,/)
  490 FORMAT (2X,4(E11.4,2X),/)
  500 FORMAT (//,5X,70H ************************ END OF CALCULATION ***
     1**********************)
      STOP
      END
C
C
C     ---------------SUBROUTINE AVEBX IS CALCULATING THE AVERAGE BETA AND
C                LAMBDA WITH THE BASIC OF SIX GROUP.
C
C                     SUPPORTING ROUTINE ,   ABETA
C                                            ALAMDA
C
C
C

      SUBROUTINE AVEBX (B,X,N,NGROUP)
      DIMENSION B(6), X(6), BX(6), XX(6), YB(6), YX(6)
      DO 10 K=1,N
      XX(K)=X(K)
   10 BX(K)=B(K)
      GO TO (20,30,60,90,130,150), NGROUP
   20 YB(NGROUP)=ABETA(B,N)
      YX(NGROUP)=ALAMDA(B,X,N)
      GO TO 150
   30 N1=NGROUP+1
      DO 50 J=1,NGROUP
      YB(J)=ABETA(B,N1)
      YX(J)=ALAMDA(B,X,N1)
      DO 40 J1=1,N1
      N2=J1+3
      X(J1)=XX(N2)
   40 B(J1)=BX(N2)
   50 CONTINUE
      GO TO 150
   60 N3=NGROUP-1
      J3=2
```

```
      DO 80 J1=1,NGROUP
      YB(J1)=ABETA(B,N3)
      YX(J1)=ALAMDA(B,X,N3)
      IF (J1.EQ.NGROUP) GO TO 150
      DO 70 J2=1,N3
      J3=J3+1
      B(J2)=BX(J3)
   70 X(J2)=XX(J3)
   80 CONTINUE
      GO TO 150
   90 N4=NGROUP-2
      J3=2
      DO 110 J1=1,N4
      YB(J1)=ABETA(B,N4)
      YX(J1)=ALAMDA(B,X,N4)
      DO 100 J2=1,N4
      J3=J3+1
      B(J2)=BX(J3)
  100 X(J2)=XX(J3)
  110 CONTINUE
      DO 120 NO=3,NGROUP
      NI=NO+2
      YB(NO)=BX(NI)
  120 YX(NO)=BX(NI)
      GO TO 150
  130 N5=NGROUP-1
      YB(1)=ABETA(B,2)
      YX(1)=ALAMDA(B,X,2)
      DO 140 J1=2,NGROUP
      J2=J1+1
      YB(J1)=BX(J2)
  140 YX(J1)=XX(J2)
      GO TO 150
  150 DO 160 J2=1,NGROUP
      B(J2)=YB(J2)
  160 X(J2)=YX(J2)
      RETURN
      END
C
C
C     --------------FUNCTION ALAMDA IS AVERAGING THE VALUE OF LAMBDA
C                 SEE EQUATION 1.3.5
C
C

      FUNCTION ALAMDA (B,X,N)
      DIMENSION B(N), X(N)
      B1=0.0
      BL=0.0
      DO 10 I=1,N
      B1=B1+B(I)
   10 BL=BL+B(I)/X(I)
      ALAMDA=B1/BL
      RETURN
      END
C
C
C     --------------FUNCTION ABETA IS AVERAGING BETA.
C                 SEE EQUATION 1.2.3
```

```
C
C

      FUNCTION ABETA (B,N)
      DIMENSION B(N)
      ABETA=0.0
      DO 10 I=1,N
   10 ABETA=ABETA+B(I)
      RETURN
      END
C
C
C     -------------SUBROUTINE U-235 IS PROVIDING DATA FOR BETA AND LAMB
C             FOR SIX GROUP DELAYED NEUTRON FROM THERMAL FISSION .
C
C

      SUBROUTINE U235 (B,X)
      DIMENSION B(6), X(6)
      B(1)=.00021
      B(2)=.00141
      B(3)=.00127
      B(4)=.00255
      B(5)=.00074
      B(6)=.00027
      X(1)=.0124
      X(2)=.0305
      X(3)=.111
      X(4)=.301
      X(5)=1.14
      X(6)=3.01
      RETURN
      END
C
C
C     --------------SUBROUTINE PU-239 IS PROVIDING DATA FOR BETA AND LAM
C             DA SIX GROUP DELAYED NEUTRON FROM THERMAL FISSION .
C
C
C

      SUBROUTINE PU239 (B,X)
      DIMENSION B(6), X(6)
      B(1)=.00009450
      B(2)=.00080460
      B(3)=.00056970
      B(4)=.00088020
      B(5)=.00023220
      B(6)=.00011880
      X(1)=.0128
      X(2)=.0301
      X(3)=.124
      X(4)=.325
      X(5)=1.12
      X(6)=2.69
      RETURN
      END
C
```

```
C
C     ------------THE MAIN PROGRAM IS TRYING TO CALCULATE THE COEFFICI
C              ENTS OF INHOUR FORMULA FOR ANY GROUP BETWEEN 1 TO 6 .
C
C
      SUBROUTINE ANHOUR (XL,X,B,R,A,NN)
      DIMENSION X(NN), B(NN), A(8)
      BB=0.0
      DO 10 I=1,NN
   10 BB=BB+B(I)
      RO=R*BB
      JJ=NN+2
      DO 20 J=1,JJ
   20 A(J)=0.0
C           SEE 1.13.13 FOR NN=1
      GO TO (30,40,50,60,70,80), NN
   30 CALL G1 (XL,X,B,RO,A)
      GO TO 90
   40 CALL G2 (XL,X,B,RO,A)
      GO TO 90
   50 CALL G3 (XL,X,B,RO,A)
      GO TO 90
   60 CALL G4 (XL,X,B,RO,A)
      GO TO 90
   70 CALL G5 (XL,X,B,RO,A)
      GO TO 90
   80 CALL G6 (XL,X,B,RO,A)
      GO TO 90
   90 RETURN
      END
C
C
C     --------------SUBROUTINE G1,G2,G3,G4,G5,G6, IS  DIRECTLY CALCULATI
C              COEFFICIENTS OF INHOUR FORMULA IN POLYNOMIAL FORM IN
C              THE ORDER OF THE SMALLEST DEGREE TO THE LARGEST .
C              THE FORM OUTPUT IS A1*W**N + A2*W**(N-1) + A3*W**(N-2)
C              +.......................................+ AN
C              SEE EQUATION 1.3.15 FOR THE GENERAL INHOUR FORMULA
C
C              SUPPORTING ROUTINE      A62,A52,A42,A32,A22  AND
C                                      XX,XY,XZ,XU,XV,AA .
C
C
C
      SUBROUTINE G1 (XL,X,B,RO,W)
      DIMENSION X(1), B(1), W(3)
      W(3)=XL
      W(2)=B(1)+XL*X(1)-RO
      W(1)=-RO*X(1)
      RETURN
      END

      SUBROUTINE G2 (XL,X,B,RO,W)
      DIMENSION X(2), B(2), W(4)
      W(4)=XL
      W(3)=XX(B,2)+XL*XX(X,2)-RO
```

```
      W(2)=XL*AA(X,2)+A62(B,X,2)-RO*XX(X,2)                             MK1 4650
      W(1)=-RO*AA(X,2)                                                  MK1 4660
      RETURN                                                            MK1 4670
      END                                                               MK1 4680

      SUBROUTINE G3 (XL,X,B,RO,W)                                       MK1 4690
      DIMENSION X(3), B(3), W(5)                                        MK1 4700
      W(5)=XL                                                           MK1 4710
      W(4)=XX(B,3)+XL*XX(X,3)-RO                                        MK1 4720
      W(3)=XL*XY(X,3)+A62(B,X,3)-RO*XX(X,3)                             MK1 4730
      W(2)=XL*AA(X,3)+A52(B,X,3)-RO*XY(X,3)                             MK1 4740
      W(1)=-RO*AA(X,3)                                                  MK1 4750
      RETURN                                                            MK1 4760
      END                                                               MK1 4770

      SUBROUTINE G4 (XL,X,B,RO,W)                                       MK1 4780
      DIMENSION X(4), B(4), W(6)                                        MK1 4790
      W(6)=XL                                                           MK1 4800
      W(5)=XX(B,4)+XL*XX(X,4)-RO                                        MK1 4810
      W(4)=XL*XY(X,4)+A62(B,X,4)-RO*XX(X,4)                             MK1 4820
      W(3)=XL*XZ(X,4)+A52(B,X,4)-RO*XY(X,4)                             MK1 4830
      W(2)=XL*AA(X,4)+A42(B,X,4)-RO*XZ(X,4)                             MK1 4840
      W(1)=-RO*AA(X,4)                                                  MK1 4850
      RETURN                                                            MK1 4860
      END                                                               MK1 4870

      SUBROUTINE G5 (XL,X,B,RO,W)                                       MK1 4880
      DIMENSION X(5), B(5), W(7)                                        MK1 4890
      W(7)=XL                                                           MK1 4900
      W(6)=XX(B,5)+XL*XX(X,5)-RO                                        MK1 4910
      W(5)=XL*XY(X,5)+A62(B,X,5)-RO*XX(X,5)                             MK1 4920
      W(4)=XL*XZ(X,5)+A52(B,X,5)-RO*XY(X,5)                             MK1 4930
      W(3)=XL*XU(X,5)+A42(B,X,5)-RO*XZ(X,5)                             MK1 4940
      W(2)=XL*AA(X,5)+A32(B,X,5)-RO*XU(X,5)                             MK1 4950
      W(1)=-RO*AA(X,5)                                                  MK1 4960
      RETURN                                                            MK1 4970
      END                                                               MK1 4980

      SUBROUTINE G6 (XL,X,B,RO,W)                                       MK1 4990
      DIMENSION X(6), B(6), W(8)                                        MK1 5000
      W(8)=XL                                                           MK1 5010
      W(7)=XX(B,6)+XL*XX(X,6)-RO                                        MK1 5020
      W(6)=XL*XY(X,6)+A62(B,X,6)-RO*XX(X,6)                             MK1 5030
      W(5)=XL*XZ(X,6)+A52(B,X,6)-RO*XY(X,6)                             MK1 5040
      W(4)=XL*XU(X,6)+A42(B,X,6)-RO*XZ(X,6)                             MK1 5050
      W(3)=XL*XV(X,6)+A32(B,X,6)-RO*XU(X,6)                             MK1 5060
      W(2)=XL*AA(X,6)+A22(B,X,6)-RO*XV(X,6)                             MK1 5070
      W(1)=-RO*AA(X,6)                                                  MK1 5080
      RETURN                                                            MK1 5090
      END                                                               MK1 5100
C                                                                       MK1 5110
C                                                                       MK1 5120
C     ---------------FUNCTION A62 FUNCTION AS :                         MK1 5130
C                   A62 = B1*(X2+X3+X4+...) + B2*(B1+B3+B4+....) +      MK1 5140
C                   B3*(B1+B2+B3+B4+...) + .....    IN THAT COMBINATIONS.MK1 5150
C                                                                       MK1 5160
```

```
      C
            FUNCTION A62 (B,X,M)
            DIMENSION X(M), AX1(6), B(M), W(6)
            A62=0.0
            DO 20 I=1,M
            DO 10 J=1,M
         10 AX1(J)=X(J)
            AX1(I)=0.0
            W(I)=B(I)*XX(AX1,M)
         20 A62=A62+W(I)
            RETURN
            END
      C
      C
      C     ---------------A52 IS ALMOST THE SAME THING AS A62 IN IT'S FUNCTION
      C            EXCEPT THAT X1 = Y1*Y2 WHICH THE POSTSCRIPT NEVER BE
      C            EQUAL AND THE FIRST CHARACTER POSTSCRIPT IS ALWAYS
      C            SMALLER THAN THE NEXT .
      C
      C
            FUNCTION A52 (B,X,M)
            DIMENSION X(M), AX2(6), B(M), W(6)
            A52=0.0
            DO 20 I=1,M
            DO 10 J=1,M
         10 AX2(J)=X(J)
            AX2(I)=0.0
            W(I)=B(I)*XY(AX2,M)
         20 A52=A52+W(I)
            RETURN
            END
      C
      C
      C     ---------------QED AS A62 EXCEPT  ITS ADDITION OF THREE CHARACTER .
      C
      C
            FUNCTION A42 (B,X,M)
            DIMENSION X(M), AX3(6), B(M), W(6)
            A42=0.0
            DO 20 I=1,M
            DO 10 J=1,M
         10 AX3(J)=X(J)
            AX3(I)=0.0
            W(I)=B(I)*XZ(AX3,M)
         20 A42=A42+W(I)
            RETURN
            END
      C
      C
      C     ---------------QED AS A62 EXCEPT CHARACTER ADDITION IS FOUR.
      C
      C
```

```
      FUNCTION A32 (B,X,M)
      DIMENSION X(M), AX4(6), B(M), W(6)
      A32=0.0
      DO 20 I=1,M
      DO 10 J=1,M
   10 AX4(J)=X(J)
      AX4(I)=0.0
      W(I)=B(I)*XU(AX4,M)
   20 A32=A32+W(I)
      RETURN
      END
C
C
C     ---------------UED AS A62 EXCEPT ADDITION CHARACTER IS FIVE .
C
C

      FUNCTION A22 (B,X,M)
      DIMENSION X(M), AX5(6), B(M), W(6)
      A22=0.0
      DO 20 I=1,M
      DO 10 J=1,M
   10 AX5(J)=X(J)
      AX5(I)=0.0
      W(I)=B(I)*XV(AX5,M)
   20 A22=A22+W(I)
      RETURN
      END
C
C
C     ---------------XX IS COMPUTING THE TOTAL OF X
C                    XX = (X1+X2+X3+X4+..........+XN)
C
C

      FUNCTION XX (X,M)
      DIMENSION X(M)
      XX=0.0
      DO 10 I=1,M
   10 XX=XX+X(I)
      RETURN
      END
C
C
C     ---------------XY IS COMPUTING THE ADDITION OF TWO PRODUCTS .
C                    XY = (X1*X2+X1*X3+X1*X4+.....+X2*X3+X2*X4+.....)
C                    THE FIRST POSTCRIPT ALWAYS SMALLER THAN THE NEXT
C
C

      FUNCTION XY (X,M)
      DIMENSION X(M)
      XY=0.0
      DO 20 I=1,M
      DO 20 J=1,M
      A=X(I)
      B=X(J)
```

```
      IF (I-J) 10,20,20
   10 Y=A*B
      XY=XY+Y
   20 CONTINUE
      RETURN
      END
C
C
C     ---------------QED AS XY EXCEPT IT IS A PRODUCT OF THREE
C
C

      FUNCTION XZ (X,M)
      DIMENSION X(M)
      XZ=0.0
      DO 30 I=1,M
      DO 30 J=I,M
      DO 30 K=1,M
      A=X(I)
      B=X(J)
      C=X(K)
      IF (I-J) 10,30,30
   10 IF (J-K) 20,30,30
   20 Y=A*B*C
      XZ=XZ+Y
   30 CONTINUE
      RETURN
      END
C
C
C     ---------------QED AS XY EXCEPT FOR PRODUCT OF FOUR
C
C

      FUNCTION XU (X,M)
      DIMENSION X(M)
      XU=0.0
      DO 40 I=1,M
      DO 40 J=1,M
      DO 40 K=1,M
      DO 40 L=1,M
      A=X(I)
      B=X(J)
      C=X(K)
      D=X(L)
      IF (I-J) 10,40,40
   10 IF (J-K) 20,40,40
   20 IF (K-L) 30,40,40
   30 Y=A*B*C*D
      XU=XU+Y
   40 CONTINUE
      RETURN
      END
C
C
C     ---------------QED EXCEPT ITS AN ADDITION OF MULTIPLICATION OF FIVE
C
C
```

```
      FUNCTION XV (X,M)
      DIMENSION X(M)
      XV=0.0
      DO 50 I=1,M
      DO 50 J=1,M
      DO 50 K=1,M
      DO 50 L=1,M
      DO 50 N=1,M
      A=X(I)
      B=X(J)
      C=X(K)
      D=X(L)
      E=X(N)
      IF (I-J) 10,50,50
   10 IF (J-K) 20,50,50
   20 IF (K-L) 30,50,50
   30 IF (L-N) 40,50,50
   40 Y=A*B*C*D*E
      XV=XV+Y
   50 CONTINUE
      WRITE (6,60) XV
   60 FORMAT (2X,7H XV  = ,F19.10)
      RETURN
      END
C
C
C     ---------------AA IS COMPUTING MULTIPLICATION OF N CHARACTER .
C           AA = X1*X2*X3*X4*..............*XN
C
C
C
      FUNCTION AA (X,M)
      DIMENSION X(M)
      AA=X(1)
      DO 10 I=2,M
   10 AA=AA*X(I)
      RETURN
      END
C
C
C     ---------------FORM N X N MATRIX BY HANSEN METHOD , G - MATRIX .
C                   THE FIRST COLUMN , THE FIRST ROW AND THE DIAGONAL # 0,
C                   THE REST OF THEM = 0 .
C                   SEE EQUATION 1.4.15
C
C                   SUPPORTING ROUTINE  NONE .
C
C
      SUBROUTINE GMTFX (R,B,X,XL,WO,N,H,G)
      DIMENSION B(6), X(6), G(N,N)
      GD(AX,H)=EXP(-AX*H)
      GH(RBL,WO,H,AX)=((EXP(WO*H)-EXP(RBL*H))/(WO-RBL))*AX
      GV(XL,WO,H,AX,BX)=((EXP(WO*H)-GD(AX,H))*BX)/((WO+AX)*XL)
      RB(R,BX)=(R-BX)/XL
      SB=0.0
      NN=N-1
      DO 10 I=1,NN
```

```
      10   BB=BB+B(I)
           DO 150 J=1,N
           DO 140 JJ=1,N
           J1=JJ
           RBL=RB(R,BB)
           IF (J-JJ) 20,60,20
      20   IF (J-1) 30,50,30
      30   J1=JJ+1
           IF (JJ-N) 50,40,50
      40   IF (J-N) 120,80,120
      50   AX=X(J1-1)
           GO TO 90
      60   IF (J-1) 80,70,80
      70   G(1,1)=EXP(RB(P,BB)*H)
           GO TO 140
      80   AX=X(J-1)
           G(J,JJ)=GD(AX,H)
           GO TO 140
      90   IF (J-1) 110,100,110
      100  G(1,JJ)=GH(RBL,WO,H,AX)
           GO TO 140
      110  IF (JJ-1) 120,130,120
      120  G(J,JJ)=0.0
           GO TO 140
      130  AX=X(J-1)
           BX=B(J-1)
           G(J,1)=GV(XL,WO,H,AX,BX)
      140  CONTINUE
           I=J
      C    WRITE(6,144)(G(I,II),II=1,N)
      C 144 FORMAT('****',2(E10.3,2X),'****')
      150  CONTINUE
           RETURN
           END
      C
      C
      C    --------------MULTIPLY  THE G - MATRIX WITH THE INITIAL VECTOR COL
      C              TO GET THE NEXT ONE , ITERATION .
      C              SEE EQUATION 1.4.3.
      C
      C

           SUBROUTINE GXN (G,XN,N,GX)
           DIMENSION G(N,N), XN(N), GX(N)
           DO 20 I=1,N
           GG=0.0
           DO 10 J=1,N
      10   GG=GG+G(I,J)*XN(J)
      20   GX(I)=GG
           RETURN
           END
      C
      C
      C
      C       SUBROUTINE POLRT
      C
      C       PURPOSE
      C          COMPUTES THE REAL AND COMPLEX ROOTS OF A REAL POLYNOMIAL
      C
```

```
C     USAGE
C        CALL POLRT(XCOF,COF,M,ROOTR,ROOTI,IER,M1)
C
C     DESCRIPTION OF PARAMETERS
C        XCOF -VECTOR OF M+1 COEFFICIENTS OF THE POLYNOMIAL
C              ORDERED FROM SMALLEST TO LARGEST POWER
C        COF  -WORKING VECTOR OF LENGTH M+1
C        M    -ORDER OF POLYNOMIAL
C        ROOTR-RESULTANT VECTOR OF LENGTH M CONTAINING REAL ROOTS
C              OF THE POLYNOMIAL
C        ROOTI-RESULTANT VECTOR OF LENGTH M CONTAINING THE
C              CORRESPONDING IMAGINARY ROOTS OF THE POLYNOMIAL
C        IER  -ERROR CODE WHERE
C              IER=0  NO ERROR
C              IER=1  M LESS THAN ONE
C              IER=2  M GREATER THAN 36
C              IER=3  UNABLE TO DETERMINE ROOT WITH 500 ITERATIONS
C                     ON 5 STARTING VALUES
C              IER=4  HIGH ORDER COEFFICIENT IS ZERO
C        M1   -NUMBER OF COEFFICIENT , M+1
C
C     REMARKS
C        LIMITED TO 36TH ORDER POLYNOMIAL OR LESS.
C        FLOATING POINT OVERFLOW MAY OCCUR FOR HIGH ORDER
C        POLYNOMIALS BUT WILL NOT AFFECT THE ACCURACY OF THE
C        RESULTS .
C
C     SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED
C        NONE
C
C     METHOD
C        NEWTON-RAPHSON ITERATIVE TECHNIQUE.  THE FINAL ITERATIONS
C        ON EACH ROOT ARE PERFORMED USING THE ORIGINAL POLYNOMIAL
C        RATHER THAN THE REDUCED POLYNOMIAL TO AVOID ACCUMULATED
C        ERRORS IN THE REDUCED POLYNOMIAL.
C
C
      SUBROUTINE POLRT (XCOF,COF,M,ROOTR,ROOTI,IER,M1)
      DIMENSION XCOF(M1), COF(M1), ROOTR(M), ROOTI(M)
C     DOUBLE PRECISION X0,Y0,X,Y,XPR,YPR,UX,UY,V,YT,XT,U,XT2,YT2,SUMSQ,
C    1 DX,DY,TEMP,ALPHA,CABS
C
C
C        IF A DOUBLE PRECISION VERSION OF THIS ROUTINE IS DESIRED, THE
C        C IN COLUMN 1 SHOULD BE REMOVED FROM THE DOUBLE PRECISION
C        STATEMENT WHICH FOLLOWS.
C
C     DOUBLE PRECISION XCOF,COF,ROOTR,ROOTI
C
C        THE C MUST ALSO BE REMOVED FROM DOUBLE PRECISION STATEMENTS
C        APPEARING IN OTHER ROUTINES USED IN CONJUNCTION WITH THIS
C        ROUTINE.
C        THE DOUBLE PRECISION VERSION MAY BE MODIFIED BY CHANGING THE
C        CONSTANT IN STATEMENT 78 TO 1.0D-12 AND IN STATEMENT 122 TO
C        1.0D-10.  THIS WILL PROVIDE HIGHER PRECISION RESULTS AT THE
C        COST OF EXECUTION TIME
C
C
```

```
      IFIT=0
      N=M
      IER=0
      IF (XCOF(N+1)) 10,40,10
   10 IF (N) 20,20,60
C
C        SET ERROR CODE TO 1
C
   20 IER=1
   30 RETURN
C
C        SET ERROR CODE TO 4
C
   40 IER=4
      GO TO 30
C
C        SET ERROR CODE TO 2
C
   50 IER=2
      GO TO 30
   60 IF (N-36) 70,70,50
   70 NX=N
      NXX=N+1
      N2=1
      KJ1=N+1
      DO 80 L=1,KJ1
      MT=KJ1-L+1
   80 COF(MT)=XCOF(L)
C
C        SET INITIAL VALUES
C
   90 X0=.00500101
      Y0=0.01000101
C
C        ZERO INITIAL VALUE COUNTER
C
      IN=0
  100 X=X0
C
C        INCREMENT INITIAL VALUES AND COUNTER
C
      X0=-10.0*Y0
      Y0=-10.0*X
C
C        SET X AND Y TO CURRENT VALUE
C
      X=X0
      Y=Y0
      IN=IN+1
      GO TO 120
  110 IFIT=1
      XPR=X
      YPR=Y
C
C        EVALUATE POLYNOMIAL AND DERIVATIVES
C
  120 ICT=0
  130 UX=0.0
      UY=0.0
      V=0.0
```

```
      YT=0.0                                                            MK1 9110
      XT=1.0                                                            MK1 9120
      U=COF(N+1)                                                        MK1 9130
      IF (U) 140,270,140                                                MK1 9140
140   DO 150 I=1,N                                                      MK1 9150
      L=N-I+1                                                           MK1 9160
      TEMP=COF(L)                                                       MK1 9170
      XT2=X*XT-Y*YT                                                     MK1 9180
      YT2=X*YT+Y*XT                                                     MK1 9190
      U=U+TEMP*XT2                                                      MK1 9200
      V=V+TEMP*YT2                                                      MK1 9210
      FI=I                                                              MK1 9220
      UX=UX+FI*XT*TEMP                                                  MK1 9230
      UY=UY-FI*YT*TEMP                                                  MK1 9240
      XT=XT2                                                            MK1 9250
150   YT=YT2                                                            MK1 9260
      SUMSQ=UX*UX+UY*UY                                                 MK1 9270
      IF (SUMSQ) 160,230,160                                            MK1 9280
160   DX=(V*UY-U*UX)/SUMSQ                                              MK1 9290
      X=X+DX                                                            MK1 9300
      DY=-(U*UY+V*UX)/SUMSQ                                             MK1 9310
      Y=Y+DY                                                            MK1 9320
      IF (ABS(DY)+ABS(DX)-1.0E-5) 210,170,170                           MK1 9330
C                                                                       MK1 9340
C        STEP ITERATION COUNTER                                         MK1 9350
C                                                                       MK1 9360
170   ICT=ICT+1                                                         MK1 9370
      IF (ICT-500) 130,180,180                                          MK1 9380
180   IF (IFIT) 210,190,210                                             MK1 9390
190   IF (IN-5) 100,200,200                                             MK1 9400
C                                                                       MK1 9410
C        SET ERROR CODE TO 3                                            MK1 9420
C                                                                       MK1 9430
200   IER=3                                                             MK1 9440
      GO TO 30                                                          MK1 9450
210   DO 220 L=1,NXX                                                    MK1 9460
      MT=KJ1-L+1                                                        MK1 9470
      TEMP=XCOF(MT)                                                     MK1 9480
      XCOF(MT)=COF(L)                                                   MK1 9490
220   COF(L)=TEMP                                                       MK1 9500
      ITEMP=N                                                           MK1 9510
      N=NX                                                              MK1 9520
      NX=ITEMP                                                          MK1 9530
      IF (IFIT) 250,110,250                                             MK1 9540
230   IF (IFIT) 240,100,240                                             MK1 9550
240   X=XPR                                                             MK1 9560
      Y=YPR                                                             MK1 9570
250   IFIT=0                                                            MK1 9580
      IF (ABS(Y)-1.0E-4*ABS(X)) 280,260,260                             MK1 9590
260   ALPHA=X+X                                                         MK1 9600
      SUMSQ=X*X+Y*Y                                                     MK1 9610
      N=N-2                                                             MK1 9620
      GO TO 290                                                         MK1 9630
270   X=0.0                                                             MK1 9640
      NX=NX-1                                                           MK1 9650
      NXX=NXX-1                                                         MK1 9660
280   Y=0.0                                                             MK1 9670
      SUMSQ=0.0                                                         MK1 9680
      ALPHA=X                                                           MK1 9690
      N=N-1                                                             MK1 9700
```

```
290   COF(2)=COF(2)+ALPHA*COF(1)                                       MK1 9710
      IF (N.EQ.0) GO TO 310                                            MK1 9720
      DO 300 L=2,N                                                     MK1 9730
300   COF(L+1)=COF(L+1)+ALPHA*COF(L)-SUMSQ*COF(L-1)                    MK1 9740
310   ROOTI(N2)=Y                                                      MK1 9750
      ROOTR(N2)=X                                                      MK1 9760
      N2=N2+1                                                          MK1 9770
      IF (SUMSQ) 320,330,320                                           MK1 9780
320   Y=-Y                                                             MK1 9790
      SUMSQ=0.0                                                        MK1 9800
      GO TO 310                                                        MK1 9810
330   IF (N) 30,30,90                                                  MK1 9820
      END                                                              MK1 9830
C                                                                      MK1 9840
C                                                                      MK1 9850
C                                                                      MK1 9860
C     --------SUBROUTINE TPLOT IS SINGLE PRECISION PLOTTING 5 DIFFERENTMK1 9870
C      VARIABLES , ASSUMING CONSTANT TIME INCREMENT BETWEEN TWO        MK1 9880
C      EVENTS .                                                        MK1 9890
C                                                                      MK1 9900
C      SUPPORTING ROUTINE NONE                                         MK1 9910
C                                                                      MK1 9920
C                                                                      MK1 9930
C                                                                      MK1 9940

      SUBROUTINE TPLOT (M1,M2,M8,M9,M0,JX)                             MK1 9950
      IMPLICIT REAL*4(A-H,M-Z)                                         MK1 9960
      DIMENSION M8(JX), M9(JX), M0(JX), M1(JX), M2(JX)                 MK1 9970
      DIMENSION LINE(61), INUM(9)                                      MK1 9980
      INTEGER PL,MI,ST,BL,SL,S9,S0,S6,S1,S2                            MK1 9990
      READ (5,80) PL,MI,ST,BL,SL,S9,S0,S1,S2,S6                        MK110000
      MXY=0.0                                                          MK110010
      MIN0=0.0                                                         MK110020
      MIN2=0.0                                                         MK110030
      MIN8=0.0                                                         MK110040
      MIN9=0.0                                                         MK110050
      PHI0=0.0                                                         MK110060
      PHI2=0.0                                                         MK110070
      PHI8=0.0                                                         MK110080
      PHI9=0.0                                                         MK110090
      DO 10 I=1,JX                                                     MK110100
      IF (MIN0.GT.M0(I)) MIN0=M0(I)                                    MK110110
      IF (MIN2.GT.M2(I)) MIN2=M2(I)                                    MK110120
      IF (MIN8.GT.M8(I)) MIN8=M8(I)                                    MK110130
      IF (MIN9.GT.M9(I)) MIN9=M9(I)                                    MK110140
      IF (ABS(M0(I)).GT.PHI0) PHI0=ABS(M0(I))                          MK110150
      IF (ABS(M2(I)).GT.PHI2) PHI2=ABS(M2(I))                          MK110160
      IF (ABS(M8(I)).GT.PHI8) PHI8=ABS(M8(I))                          MK110170
      IF (ABS(M9(I)).GT.PHI9) PHI9=ABS(M9(I))                          MK110180
10    CONTINUE                                                         MK110190
      JJ=JX                                                            MK110200
      JJ0=JJ/6+1                                                       MK110210
      JJ1=JJ+1                                                         MK110220
      WRITE (6,90)                                                     MK110230
      WRITE (6,100)                                                    MK110240
      PHI0=PHI0+ABS(MIN0)                                              MK110250
      PHI2=PHI2+ABS(MIN2)                                              MK110260
      PHI8=PHI8+ABS(MIN8)                                              MK110270
      PHI9=PHI9+ABS(MIN9)                                              MK110280
```

```
      DO 20 I=1,JJ
      IF (MINO.LT.0.0) M0(I)=M0(I)+ABS(MINO)
      IF (MIN2.LT.0.0) M2(I)=M2(I)+ABS(MIN2)
      IF (MIN8.LT.0.0) M8(I)=M8(I)+ABS(MIN8)
      IF (MIN9.LT.0.0) M9(I)=M9(I)+ABS(MIN9)
      M0(I)=M0(I)/PHI0
      M2(I)=M2(I)/PHI2
      M8(I)=M8(I)/PHI8
   20 M9(I)=M9(I)/PHI9
      DO 30 I=1,9
   30 INUM(I)=I
      WRITE (6,110) (INUM(I),I=1,9)
      DO 70 I=1,JJ1
      IF (I.EQ.1) GO TO 50
      MXY=M1(I-1)
      IP8=M8(I-1)*60+1.0
      IP9=M9(I-1)*60+1.0
      IP0=M0(I-1)*60+1.0
      IP2=M2(I-1)*60+1.0
      DO 40 I1=1,56,5
      LINE(I1)=BL
      DO 40 I2=1,5
      I3=I1+I2
      IF (I1.EQ.IP0) LINE(I1)=S0
      IF (I3.EQ.IP0) LINE(I3)=S0
      IF (I1.EQ.IP2) LINE(I1)=S2
      IF (I3.EQ.IP2) LINE(I3)=S2
      IF (I1.EQ.IP8) LINE(I1)=S1
      IF (I3.EQ.IP8) LINE(I3)=S1
      IF (I1.EQ.IP9) LINE(I1)=S9
      IF (I3.EQ.IP9) LINE(I3)=S9
   40 CONTINUE
      LINE(61)=PL
      IIL=I-1
      IF(I .EQ.IP0) LINE(I)=S0
      IF(I .EQ.IP2) LINE(I)=S2
      IF(I .EQ.IP8) LINE(I)=S1
      IF(I .EQ.IP9) LINE(I)=S9
      IF(IP0.EQ.61)LINE(61)=S0
      IF(IP2.EQ.61)LINE(61)=S2
      IF(IP8.EQ.61)LINE(61)=S1
      IF(IP9.EQ.61)LINE(61)=S9
      IF(IP9.NE.1.OR.IP8.NE.1.OR.IP0.NE.1.OR.IP2.NE.1)LINE(1)=PL
      WRITE(6,120)MXY,(LINE(KK),KK=1,61)
      IF(I.EQ.JJ1) GO TO 70
   50 CONTINUE
      DO 60 I1=1,56,5
      DO 60 I2=1,5
      I3=I1+I2
      LINE(I3)=BL
   60 CONTINUE
   70 CONTINUE
      WRITE(6,130)(INUM(I),I=1,9)
      WRITE(6,150)PL,MI,ST,BL,SL,S9,S0,S1,S2,S6
      WRITE(6,160)
      WRITE (6,140)
   80 FORMAT(11A1)
   90 FORMAT('1')
  100 FORMAT(35X,'RELATIVE DENSITY')
  110 FORMAT (17X,9(2X,'0.',I1,' ')/,14X,10('+-----'),'+')
```

```
      120  FORMAT(1X,'TIME',1X,F7.3,1X,61A1)                                MK110890
      130  FORMAT(14X,10('+-----'),'+',/,16X,9(3X,'0.',I1))                 MK110900
      140  FORMAT ('1').                                                    MK110910
      150  FORMAT(15X,'INPUT CHARACTER  ',11A1,/)                           MK110920
      160  FORMAT('                         ',/,15X,' P - POWER',/,15X,' L -MK110930
          < LOG. OF POWER',/,15X,' Q - PRECURSOR DENSITY',/,15X,' R - REACTIVMK110940
          <ITY',///,16X,'NEGATIVE VALUE PLOTTED WITH THE AXIS IN THE CENTER',MK110950
          <////)                                                            MK110960
           STOP                                                             MK110970
           END                                                              MK110980

//DATA
```

******************************** KINETICS MODULE 1 ********************

****
INPUT PARAMETER
****

TYPE OF FUEL U-239

NUMBER OF DELAYED NEUTRON = 3

NEUTRON GENERATION TIME = 0.00010 (SEC)

TYPE OF REACTIVITY = 2

RO = $ 0.250 B2 = 5.00000

TOTAL TIME USED = 1.000 (SEC) , TIME INTERVAL = 0.010 (SEC)

OUTPUT OPTION = 2

INITIAL RELATIVE FLUX = 1.000

********* END OF INPUT DATA **********

SIX GROUP LAMBDA ARE 0.0128 0.0301 0.1240 0.3250 1.1200 2.6900

SIX GROUP BETA ARE 0.00009 0.00080 0.00057 0.00088 0.00023 0.00012

THE AVERAGE BETA AND LAMDA

B(1)=	0.00090	LAMBDA(1) =	0.02636
B(2)=	0.00145	LAMBDA(2) =	0.19854
B(3)=	0.00035	LAMBDA(3) =	1.39571

COEFFICIENTS OF INHOUR FORMULA

A(1) = -0.0000E 00

A(2) = 0.3051E-03

A(3) = 0.3606E-02

A(4) = 0.2862E-02

A(5) = 0.1000E-03

THE LARGEST EIGEN-VALUE = 0.0000E 00

NO ( )	TIME (SEC)	R ($)	POWER (-)	3 GROUP DELAYED NEUTRON (RELATIVE)		
1	0.000	0.00	0.1000E 01	0.3411E 03	0.7303E 02	0.2515E 01

THE G-MATRIX

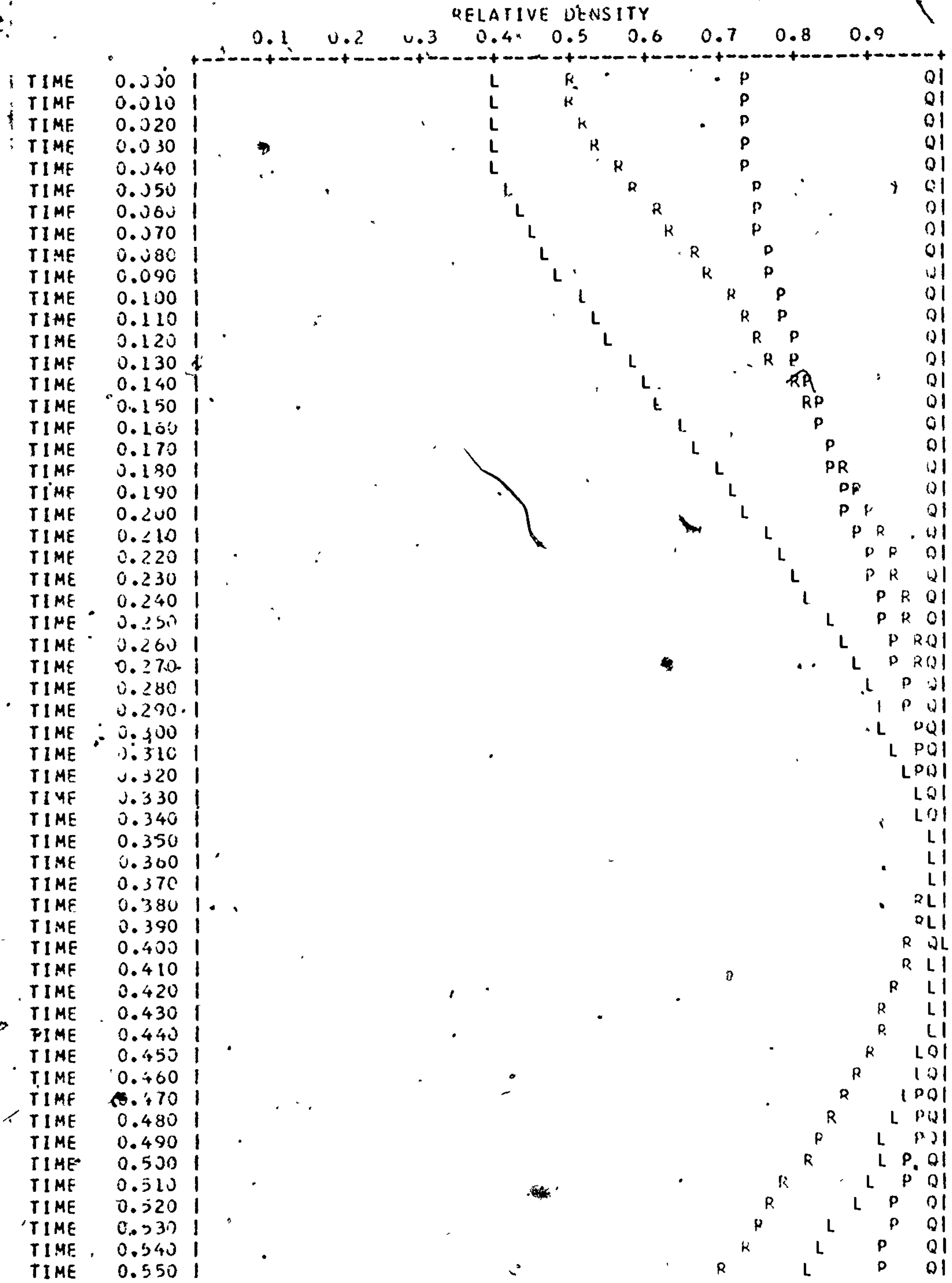
0.7634E 00	0.2310E-03	0.1740E-02	0.1223E-01
0.8991E-01	0.9997E 00	0.0000E 00	0.0000E 00
0.1448E 00	0.0000E 00	0.9980E 00	0.0000E 00
0.3486E-01	0.0000E 00	0.0000E 00	0.9861E 00

2	0.000	0.00	0.1000E 01	0.3411E 03	0.7303E 02	0.2515E 01
3	0.010	0.01	0.1003E 01	0.3411E 03	0.7303E 02	0.2515E 01
4	0.020	0.02	0.1008E 01	0.3411E 03	0.7303E 02	0.2515E 01
5	0.030	0.04	0.1015E 01	0.3411E 03	0.7303E 02	0.2515E 01
6	0.040	0.05	0.1024E 01	0.3411E 03	0.7303E 02	0.2516E 01
7	0.050	0.06	0.1033E 01	0.3411E 03	0.7303E 02	0.2517E 01
8	0.060	0.07	0.1043E 01	0.3411E 03	0.7304E 02	0.2518E 01
9	0.070	0.09	0.1055E 01	0.3411E 03	0.7304E 02	0.2519E 01
10	0.080	0.10	0.1066E 01	0.3411E 03	0.7305E 02	0.2521E 01
11	0.090	0.11	0.1078E 01	0.3412E 03	0.7306E 02	0.2523E 01
12	0.100	0.12	0.1091E 01	0.3412E 03	0.7307E 02	0.2526E 01
13	0.110	0.13	0.1103E 01	0.3412E 03	0.7309E 02	0.2529E 01
14	0.120	0.14	0.1116E 01	0.3412E 03	0.7310E 02	0.2532E 01
15	0.130	0.15	0.1129E 01	0.3412E 03	0.7312E 02	0.2536E 01
16	0.140	0.16	0.1143E 01	0.3412E 03	0.7314E 02	0.2540E 01
17	0.150	0.17	0.1156E 01	0.3412E 03	0.7316E 02	0.2545E 01
18	0.160	0.18	0.1169E 01	0.3412E 03	0.7318E 02	0.2550E 01
19	0.170	0.19	0.1182E 01	0.3412E 03	0.7320E 02	0.2555E 01
20	0.180	0.20	0.1195E 01	0.3413E 03	0.7323E 02	0.2561E 01
21	0.190	0.20	0.1207E 01	0.3413E 03	0.7326E 02	0.2567E 01
22	0.200	0.21	0.1220E 01	0.3413E 03	0.7329E 02	0.2574E 01
23	0.210	0.22	0.1232E 01	0.3413E 03	0.7332E 02	0.2581E 01
24	0.220	0.22	0.1244E 01	0.3413E 03	0.7335E 02	0.2588E 01
25	0.230	0.23	0.1255E 01	0.3414E 03	0.7338E 02	0.2595E 01
26	0.240	0.23	0.1266E 01	0.3414E 03	0.7342E 02	0.2603E 01
27	0.250	0.24	0.1276E 01	0.3414E 03	0.7346E 02	0.2611E 01

28	0.260	0.24	0.1286E 01	0.3414E 03	0.7350E 02	0.2619E 01
29	0.270	0.24	0.1295E 01	0.3414E 03	0.7354E 02	0.2628E 01
30	0.280	0.25	0.1303E 01	0.3415E 03	0.7358E 02	0.2637E 01
31	0.290	0.25	0.1311E 01	0.3415E 03	0.7362E 02	0.2646E 01
32	0.300	0.25	0.1318E 01	0.3415E 03	0.7367E 02	0.2655E 01
33	0.310	0.25	0.1324E 01	0.3416E 03	0.7371E 02	0.2664E 01
34	0.320	0.25	0.1329E 01	0.3416E 03	0.7376E 02	0.2673E 01
35	0.330	0.25	0.1333E 01	0.3416E 03	0.7380E 02	0.2682E 01
36	0.340	0.25	0.1336E 01	0.3416E 03	0.7385E 02	0.2691E 01
37	0.350	0.25	0.1338E 01	0.3417E 03	0.7390E 02	0.2701E 01
38	0.360	0.24	0.1339E 01	0.3417E 03	0.7394E 02	0.2710E 01
39	0.370	0.24	0.1339E 01	0.3417E 03	0.7399E 02	0.2719E 01
40	0.380	0.24	0.1338E 01	0.3418E 03	0.7404E 02	0.2728E 01
41	0.390	0.23	0.1336E 01	0.3418E 03	0.7409E 02	0.2737E 01
42	0.400	0.23	0.1333E 01	0.3418E 03	0.7413E 02	0.2746E 01
43	0.410	0.22	0.1329E 01	0.3419E 03	0.7418E 02	0.2754E 01
44	0.420	0.22	0.1324E 01	0.3419E 03	0.7422E 02	0.2762E 01
45	0.430	0.21	0.1318E 01	0.3419E 03	0.7427E 02	0.2770E 01
46	0.440	0.20	0.1311E 01	0.3419E 03	0.7431E 02	0.2778E 01
47	0.450	0.19	0.1303E 01	0.3420E 03	0.7435E 02	0.2785E 01
48	0.460	0.19	0.1295E 01	0.3420E 03	0.7439E 02	0.2792E 01
49	0.470	0.18	0.1285E 01	0.3420E 03	0.7443E 02	0.2798E 01
50	0.480	0.17	0.1275E 01	0.3420E 03	0.7447E 02	0.2804E 01
51	0.490	0.16	0.1264E 01	0.3421E 03	0.7451E 02	0.2810E 01
52	0.500	0.15	0.1253E 01	0.3421E 03	0.7455E 02	0.2815E 01
53	0.510	0.14	0.1240E 01	0.3421E 03	0.7458E 02	0.2819E 01
54	0.520	0.13	0.1228E 01	0.3421E 03	0.7461E 02	0.2824E 01
55	0.530	0.12	0.1215E 01	0.3422E 03	0.7464E 02	0.2827E 01
56	0.540	0.11	0.1201E 01	0.3422E 03	0.7467E 02	0.2830E 01
57	0.550	0.10	0.1188E 01	0.3422E 03	0.7469E 02	0.2833E 01
58	0.560	0.08	0.1174E 01	0.3422E 03	0.7472E 02	0.2835E 01
59	0.570	0.07	0.1160E 01	0.3422E 03	0.7474E 02	0.2837E 01
60	0.580	0.06	0.1145E 01	0.3422E 03	0.7476E 02	0.2838E 01
61	0.590	0.05	0.1131E 01	0.3422E 03	0.7478E 02	0.2838E 01
62	0.600	0.04	0.1117E 01	0.3423E 03	0.7479E 02	0.2838E 01
63	0.610	0.02	0.1102E 01	0.3423E 03	0.7481E 02	0.2838E 01
64	0.620	0.01	0.1088E 01	0.3423E 03	0.7482E 02	0.2837E 01
65	0.630	-0.00	0.1074E 01	0.3423E 03	0.7483E 02	0.2836E 01
66	0.640	-0.01	0.1060E 01	0.3423E 03	0.7483E 02	0.2834E 01
67	0.650	-0.03	0.1046E 01	0.3423E 03	0.7484E 02	0.2832E 01
68	0.660	-0.04	0.1033E 01	0.3423E 03	0.7484E 02	0.2829E 01
69	0.670	-0.05	0.1020E 01	0.3423E 03	0.7484E 02	0.2826E 01
70	0.680	-0.06	0.1007E 01	0.3423E 03	0.7484E 02	0.2822E 01
71	0.690	-0.08	0.9947E 00	0.3423E 03	0.7484E 02	0.2818E 01
72	0.700	-0.09	0.9826E 00	0.3423E 03	0.7483E 02	0.2814E 01
73	0.710	-0.10	0.9708E 00	0.3423E 03	0.7483E 02	0.2809E 01
74	0.720	-0.11	0.9595E 00	0.3423E 03	0.7482E 02	0.2804E 01
75	0.730	-0.12	0.9485E 00	0.3423E 03	0.7481E 02	0.2798E 01
76	0.740	-0.13	0.9380E 00	0.3423E 03	0.7480E 02	0.2793E 01
77	0.750	-0.14	0.9279E 00	0.3423E 03	0.7479E 02	0.2787E 01
78	0.760	-0.15	0.9182E 00	0.3423E 03	0.7477E 02	0.2780E 01
79	0.770	-0.16	0.9089E 00	0.3423E 03	0.7476E 02	0.2774E 01
80	0.780	-0.17	0.9001E 00	0.3423E 03	0.7474E 02	0.2767E 01
81	0.790	-0.18	0.8918E 00	0.3422E 03	0.7472E 02	0.2760E 01
82	0.800	-0.19	0.8838E 00	0.3422E 03	0.7471E 02	0.2753E 01
83	0.810	-0.20	0.8764E 00	0.3422E 03	0.7468E 02	0.2745E 01
84	0.820	-0.20	0.8693E 00	0.3422E 03	0.7466E 02	0.2738E 01
85	0.830	-0.21	0.8627E 00	0.3422E 03	0.7464E 02	0.2730E 01
86	0.840	-0.22	0.8566E 00	0.3422E 03	0.7462E 02	0.2723E 01
87	0.850	-0.22	0.8509E 00	0.3422E 03	0.7459E 02	0.2715E 01

88	0.860	-0.23	0.8456E 00	0.3422E 03	0.7457E 02	0.2707E 01
89	0.870	-0.23	0.8408E 00	0.3421E 03	0.7454E 02	0.2699E 01
90	0.880	-0.24	0.8364E 00	0.3421E 03	0.7452E 02	0.2691E 01
91	0.890	-0.24	0.8324E 00	0.3421E 03	0.7449E 02	0.2682E 01
92	0.900	-0.24	0.8288E 00	0.3421E 03	0.7446E 02	0.2674E 01
93	0.910	-0.25	0.8257E 00	0.3421E 03	0.7444E 02	0.2666E 01
94	0.920	-0.25	0.8230E 00	0.3421E 03	0.7441E 02	0.2658E 01
95	0.930	-0.25	0.8207E 00	0.3421E 03	0.7438E 02	0.2650E 01
96	0.940	-0.25	0.8189E 00	0.3420E 03	0.7435E 02	0.2642E 01
97	0.950	-0.25	0.8174E 00	0.3420E 03	0.7432E 02	0.2634E 01
98	0.960	-0.25	0.8164E 00	0.3420E 03	0.7429E 02	0.2626E 01
99	0.970	-0.25	0.8158E 00	0.3420E 03	0.7426E 02	0.2618E 01
100	0.980	-0.25	0.8155E 00	0.3420E 03	0.7424E 02	0.2610E 01

************************** END OF CALCULATION **************************


RELATIVE DENSITY
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TIME 0.010
TIME 0.020
TIME 0.030
TIME 0.040
TIME 0.050
TIME 0.060
TIME 0.070
TIME 0.080
TIME 0.090
TIME 0.100
TIME 0.110
TIME 0.120
TIME 0.130
TIME 0.140
TIME 0.150
TIME 0.160
TIME 0.170
TIME 0.180
TIME 0.190
TIME 0.200
TIME 0.210
TIME 0.220
TIME 0.230
TIME 0.240
TIME 0.250
TIME 0.260
TIME 0.270
TIME 0.280
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TIME 0.310
TIME 0.320
TIME 0.330
TIME 0.340
TIME 0.350
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TIME 0.370
TIME 0.380
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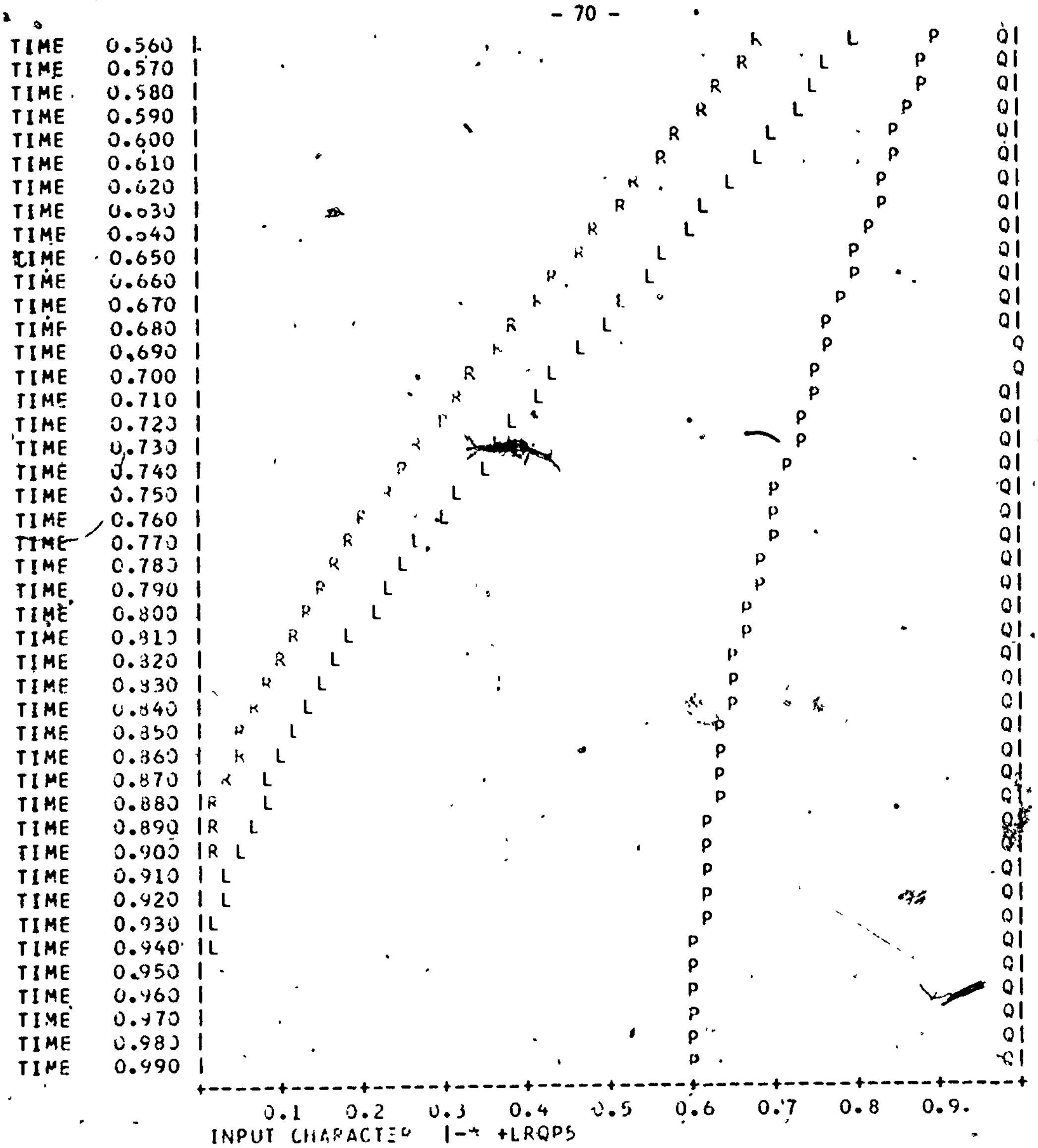


INPUT CHARACTER |-* +LRQPS

P - POWER
L - LOG. OF POWER
Q - PRECURSOR DENSITY
R - REACTIVITY

NEGATIVE VALUE PLOTTED WITH THE AXIS IN THE CENTER

REACTOR DYNAMICS MODULE, RD-2

REACTOR KINETICS WITH FEEDBACK

by

Ronald J. Onega

The University gratefully acknowledges the support of the Division of Higher Education of the National Science Foundation for support of this work performed under Grant GZ-2888 and the support of Duke Power Company, North Carolina Power and Light Company, and Virginia Electric and Power Company.

Project Director: Milton C. Edlund

ACKNOWLEDGMENT

The author would like to extend his appreciation and thanks to Mr. Tjeri Surjanto who did the programming for this module.

TABLE OF CONTENTS

REACTOR DYNAMICS MODULE, RD-2

REACTOR KINETICS WITH FEEDBACK

2.1 Object of Module

The object of this module is to:

(1) Examine the temperature feedback mechanism of a PWR and

(2) Solve the one delayed neutron model with temperature feedback for a step insertion and a ramp insertion of reactivity.

The time dependence of a reactor, taking the feedback mechanisms into account, is relatively difficult. We will consider a PWR core with a two path feedback. The reactivity is diminished as the temperature of the fuel increases due to the Doppler broadening of the resonances. This feedback is instantaneous since the temperature increase follows the power generated immediately. The second feedback path is that of the moderator temperature coefficient. As the moderator temperature increases, the number density decreases and the neutron mean free path increases so that leakage increases and reactivity decreases.

We will be concerned about the stability of the reactor to a limited degree. The dynamic response depends upon the magnitude of the temperature coefficients as well as that of the signs. For a given reactor design, i.e., a given lifetime [illegible] power level, the reactor may or may not be stable for a given set of reactivity coefficients.

The core region is the only one of interest in this module. The rest of the primary loop as well as the secondary loop is treated in an overall dynamics module for a PWR.

Also, all of our analysis will be fundamental mode analysis. The physical phenomena are taking place so slowly that the higher harmonics of the flux distribution are all dying out so rapidly that we only need to consider the lowest or fundamental mode.

The thermal analysis really should proceed by the solution of the space-time heat conduction equation. This is a very complicated procedure and would also mean that spatial effects of the kinetics equations should be taken into account. We, therefore, will assume only a lumped parameter model and will obtain the time dependence of a reactor which is really one with the average properties of the reactor under consideration.

The program name is FUMOTEM which is an acronym for "Fundamental Mode Kinetics with Temperature Feedback."

There are four types of reactivity inputs that the program can accommodate with NRO = 1, 2, 3 or 4 respectively:

1) $\rho_o(t) = \rho_o \qquad o \leq t \leq t_r$
$\qquad\quad = o \qquad$ otherwise

2) $\rho_o(t) = \rho_o(1 + a\ t) \qquad o \leq t \leq t_r$
$\qquad\quad = \rho_o(1 + a\ t_r) \qquad$ otherwise

3) $\rho_o(t) = \rho_o \cos a\ t$

or

4) $\rho_o(t) = \rho_o \sin a\ t$

The feedback reactivity is taken to be zero for times $t > t_r$. One can feed in a t_r greater than the calculation time however. So for $t > t_r$, the only reactivity is due to feedback effects.

2.2 The Feedback Model

We will use the one group delayed neutron model to describe the core neutronics. The kinetics equations are

$$\frac{dP(t)}{dt} = \frac{\rho(t) - \beta}{\Lambda} P(t) + \lambda\, Q(t) \tag{2.2.1}$$

and

$$\frac{dQ(t)}{dt} = \frac{\beta}{\Lambda} P(t) - \lambda\, Q(t) \tag{2.2.2}$$

where

$P(t)$ = The total reactor power (Megawatts)

$Q(t)$ = The power equivalence of the delayed neutron precursors (Megawatts).

Now we let ΔT_M be the deviation of the spatially averaged moderator temperature from its equilibrium value, i.e.,

$$\Delta T_M(t) = T_M(t) - T_{MO} \tag{2.2.3}$$

and similarly for the fuel temperature T_F we have

$$\Delta T_F(t) = T_F(t) - T_{FO} \tag{2.2.4}$$

where T_{MO} and T_{FO} are the equilibrium moderator and fuel temperatures respectively. Also, we let α_M and α_F be the moderator and fuel temperature coefficients of reactivity. Then (1)

$$\rho(t) = \rho_o(t) + \alpha_M \Delta T_M(t) + \alpha_F \Delta_F(t) \tag{2.2.5}$$

where generally, α_M and α_F will be negative or at least their sum is negative.

The temperature coefficient of reactivity for the moderator is

$$\alpha_M \equiv \frac{\partial\rho}{\partial T_M} \simeq \frac{1}{k_{eff}} \frac{\partial k_{eff}}{\partial T_M} \tag{2.2.6}$$

and

$$\alpha_F = \frac{\partial\rho}{\partial T_{F_o}} \simeq \frac{1}{k_{eff}} \frac{\partial k_{eff}}{\partial T_F} \tag{2.2.7}$$

The thermal analysis of the core must now be considered and connected to the neutronics. The heat generated depends upon the fission rate or the power. We will look at the fuel temperature averaged over the core as well as an averaged coolant temperature. We will ignore the cladding of the fuel pins.

The heat balance equation for reactor fuel is

$$\begin{bmatrix}\text{Rate at which the}\\ \text{internal energy of}\\ \text{the fuel changes}\end{bmatrix} = \begin{bmatrix}\text{Production rate}\\ \text{of the energy}\\ \text{in the fuel}\end{bmatrix} - \begin{bmatrix}\text{Rate at which}\\ \text{heat is conducted}\\ \text{out of the fuel}\end{bmatrix}$$

or

$$\rho_F C_F V_F \frac{dT_F(t)}{dt} = P(t) - 4\pi k_F L_F (T_o - T_R) N \tag{2.2.8}$$

where

ρ_F = density of fuel (lb/ft^3)

C_F = specific heat of fuel (Btu/lb-°F)

V_F = volume of fuel (ft^3)

T_F = average temperature of the fuel (°F)

$P(t)$ = total power of reactor (Btu/hr)

k_F = thermal conductivity of fuel $(\frac{Btu}{hr\text{-}ft\text{-}°F})$

L_F = length of fuel pin (ft)

T_o = centerline temperature of the fuel (°F)

T_R = temperature of the fuel pellets at the outer edge (pellet - water interface) (°F)

N = total number of fuel pins in the reactor.

The expression for Equation (2.2.8) was obtained from El-Wakil, "Nuclear Heat Transport", page 123, equation 5-48.

The temperatures are averaged over the fuel pins. If we number each of the pins in the core, then the centerline temperature is

$$T_0 = \frac{1}{N} (T_{01} + T_{02} + \dots + T_{0N})$$

where T_{0i} is the centerline temperature for the ith pin. T_F is defined similarly so that it is the spatially averaged temperature of the "average fuel pin."

Equation (2.2.8) contains the centerline temperature and the temperature at the edge of the pellet T_R which must be eliminated. We assume that a parabolic temperature distribution holds even in the transient situation (really the transient heat conduction equation holds here) so that for one fuel pin we have

$$T(r) = T_o - \frac{P(t)r^2}{4k_F V_F} \qquad (2.2.9)$$

where T(r) is the temperature of the fuel pin a distance r from the center of the pin. The average fuel temperature is then

$$T_F(t) = \frac{2\pi}{\pi R_F^2} \int_0^{R_F} rdr \left[T_o - \frac{P(t) r^2}{4k_F V_F} \right]$$

$$= T_o - \frac{P(t)}{8k_F V_F} R_F^2. \qquad (2.2.10)$$

Eliminating the centerline temperatures, we have, using (2.2.10),

$$T_F(t) = T_R(t) + \frac{R_F^2 P(t)}{8k_F V_F}. \qquad (2.2.11)$$

Equation (2.2.8) can now be written as

$$\rho_F C_F V_F \frac{dT_F(t)}{dt} = P(t) - 4\pi k_F L_F N \left[T_F(t) + \frac{P(t) R_F^2}{8k_F V_F} - T_R(t) \right]$$

$$= (1/2) P(t) - 4\pi k_F N L_F T_F(t) + 4\pi k_F L_F T_R(t) \qquad (2.2.12)$$

The wall temperature of the pellets $T_R(t)$ is connected to the coolant temperature $T_M(t)$ since

$$T_R(t) - T_M(t) = \frac{P(t)}{h_T A_F} \qquad (2.2.13)$$

where A_F is the total area of the fuel and h_T is the heat transfer coefficient for the fuel water interface (Btu/(Ft^2-hr-°F)).

The energy balance for the water in the core is

[Heat stored in water in core] = [Heat conducted in from fuel] - [Heat transferred out of reactor core]

or mathematically,

$$\rho_M C_M V_M \frac{dT_M(t)}{dt} = 2\pi R_F L_F h_T N[T_R(t) - T_M(t)] + \dot{m}_M[C_{M1}T_{M1} - C_{M2}T_{M2}] \qquad (2.2.14)$$

where $\dot{m}_M$ is the mass flow rate of the water through the core and $T_M(t)$ is the average moderator temperature. Also, assume $C_{M1} = C_{M2}$.

We will assume $\dot{m}_M$ is an input, as is T_{M1}, the moderator inlet temperature. The moderator outlet temperature is related to the inlet and average temperatures as

$$T_{M2}(t) = 2T_M(t) - T_{M1}$$

so that Equation (2.2.14) becomes (using Equation (2.2.13))

$$\rho_M C_M V_M \frac{dT_M(t)}{dt} = 2\pi R_F L_F h_T \cdot [\frac{P(t)}{h_T A_F}] + 2\dot{m}_M C_M [T_{M1} - T_M(t)]$$

$$= P(t) + 2\dot{m}_M C_M T_{M1} - 2\dot{m}_M C_M T_M(t) \qquad (2.2.15)$$

This can be rewritten as

$$\frac{dT_M(t)}{dt} = \frac{P(t)}{\rho_M C_M V_M} - \frac{2\dot{m}_M}{\rho_M V_M} \cdot T_M(t) + \frac{2\dot{m}_M}{\rho_M V_M} T_{M1} \qquad (2.2.16)$$

The area of the fuel is $2NL_F\pi R_F$ and the fuel volume is $\pi R_F^2 L_F N$.

In summary, the equations we must solve are Equations (2.2.1), (2.2.2), (2.2.12) and (2.2.16), the last two of which are written

$$\rho_F C_F V_F \frac{dT_F}{dt} = (1/2)P(t) - 4\pi k_F L_F N T_F(t) + 4\pi k_F L_F N T_M(t), \qquad (2.2.17)$$

and

$$\frac{dT_M(t)}{dt} = \frac{P(t)}{\rho_M C_M V_M} - \frac{2\dot{m}_M}{\rho_M V_M} T_M(t) + \frac{2}{\rho_M V_M} \dot{m}_M T_{M1} \qquad (2.2.18)$$

If we put these equations into matrix form we have

$$\frac{d}{dt}\begin{bmatrix} P(t) \\ Q(t) \\ T_F(t) \\ T_M(t) \end{bmatrix} = \begin{bmatrix} \frac{\rho-\beta}{\Lambda} & \lambda & 0 & 0 \\ \frac{\beta}{\Lambda} & -\lambda & 0 & 0 \\ \frac{1}{2\,\rho_F C_F V_F} & 0 & -\frac{4\pi K_F L_F N}{\rho_F C_F V_F} & \frac{4\pi K_F L_F N}{\rho_F C_F V_F} \\ \frac{1}{\rho_M C_M V_M} & 0 & 0 & -\frac{2\dot{m}_M}{\rho_M V_M} \end{bmatrix} \times \begin{bmatrix} P(t) \\ Q(t) \\ T_F(t) \\ T_M(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{2}{\rho_M V_M}\dot{m}_M T_{M1}(t) \end{bmatrix}, \qquad (2.2.19)$$

or

$$\frac{d\,\underline{\phi}(t)}{dt} = \underline{\underline{A}}\,\underline{\phi} + \underline{B} \qquad (2.2.20)$$

where the $\underline{\underline{A}}$ is dependent upon the temperatures themselves. Thus, Equation (2.2.20) is non-linear.

The linearization of Equation (2.2.19) (or (2.2.20)) can be achieved assuming that we can look at changes about some operating point. Expanding about the operating point we have

$$P(t) = P^\circ + \Delta P \tag{2.2.21}$$

$$Q(t) = Q^\circ + \Delta Q \tag{2.2.22}$$

$$T_F(t) = T_F^\circ + \Delta T_F \tag{2.2.23}$$

$$T_M(t) = T_M^\circ + \Delta T_M \tag{2.2.24}$$

We assume that P° etc. are independent of time so

$$\frac{d\ \Delta\underline{\phi}}{dt} = (\underline{\underline{A}}^\circ + \Delta\underline{\underline{A}})\ (\underline{\phi}^\circ + \Delta\underline{\phi}) + \underline{B}$$

or neglecting the $\Delta\underline{\underline{A}} \cdot \Delta\underline{\phi}$ term we have

$$\frac{d}{dt}\Delta\underline{\phi} = \underline{\underline{A}}^\circ\ \underline{\phi}^\circ + \underline{\underline{A}}^\circ\ \Delta\underline{\phi} + \Delta\underline{\underline{A}}\ \underline{\phi}^\circ + \underline{B}$$

or

$$\frac{d}{dt}\Delta\underline{\phi} = \underline{\underline{A}}^\circ\ \Delta\underline{\phi} + \Delta\underline{\underline{A}}\underline{\phi}^\circ \tag{2.2.25}$$

since

$$\underline{\underline{A}}^\circ\ \underline{\phi}^\circ + \underline{B} = \underline{0}.$$

Writing Equation (2.2.25) out explicitly, we have

$$\frac{d}{dt}\begin{bmatrix}\Delta P\\ \Delta Q\\ \Delta T_F\\ \Delta T_M\end{bmatrix} = \begin{bmatrix}\frac{\rho_o-\beta}{\Lambda} & \lambda & 0 & 0\\ \frac{\beta}{\Lambda} & -\lambda & 0 & 0\\ \frac{1}{2\rho_F C_F V_F} & 0 & \frac{-4\pi k_F L_F N}{\rho_F C_F V_F} & \frac{4\pi k_F L_F N}{\rho_F C_F V_F}\\ \frac{1}{\rho_M C_M V_M} & 0 & 0 & \frac{-2\dot{m}_M}{\rho_M V_M}\end{bmatrix}\begin{bmatrix}\Delta P\\ \Delta Q\\ \Delta T_F\\ \Delta T_M\end{bmatrix}$$

$$+\begin{bmatrix}\frac{\alpha_F\Delta T_F+\alpha_M\Delta T_M}{\Lambda} & 0 & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\end{bmatrix}\begin{bmatrix}P^\circ\\ Q^\circ\\ T_F^\circ\\ T_M^\circ\end{bmatrix}$$

or

$$\frac{d}{dt}\begin{bmatrix}\Delta P\\ \Delta Q\\ \Delta T_F\\ \Delta T_M\end{bmatrix} = \begin{bmatrix}\frac{\rho_o-\beta}{\Lambda} & \lambda & \frac{\alpha_F P^\circ}{\Lambda} & \frac{\alpha_M P^\circ}{\Lambda}\\ \frac{\beta}{\Lambda} & -\lambda & 0 & 0\\ \frac{1}{2\rho_F C_F V_F} & 0 & \frac{-4\pi k_F L_F N}{\rho_F C_F V_F} & \frac{4\pi k_F L_F N}{\rho_F C_F V_F}\\ \frac{1}{\rho_M V_M C_M} & 0 & 0 & \frac{-2\dot{m}_M}{\rho_M V_M}\end{bmatrix}\begin{bmatrix}\Delta P\\ \Delta Q\\ \Delta T_F\\ \Delta T_M\end{bmatrix} \quad (2.2.26)$$

Equation (2.2.26) is a linearized version of Equation (2.2.19). This equation is soluble by the ordinary method of eigenvalues. In matrix form, this can be written as

$$\frac{d\Delta \underline{\phi}}{dt} = \underline{\underline{A}}' \, \underline{\Delta\phi}, \tag{2.2.27}$$

and $\underline{\underline{A}}'$ is given in terms of the equilibrium values.

The system of Equations (2.2.19) is highly non-linear because of the first element of the matrix $\underline{\underline{A}}$. The method discussed in Kinetics Module 1, developed by Hansen, is still applicable even to non-linear systems. There is a slight modification that we must discuss and this will be outlined in a later section.

The overall heat transfer coefficient is also relatively difficult to obtain so we discuss this and related parameters in the next section.

Problem 2.2.1

Show that if T_o is the temperature in the center of a fuel pin and T_M is the moderator temperature, then

$$T_o \doteq T_M + \frac{P(t)a^2}{4k_F V_F} + \frac{P(t)}{2V_F}\left[\frac{1}{k\ \text{clad}}\,\ell n\,\frac{b}{a} + \frac{1}{h_T b}\right].$$

The inner radius of the clad is "a" and the outer radius is "b".

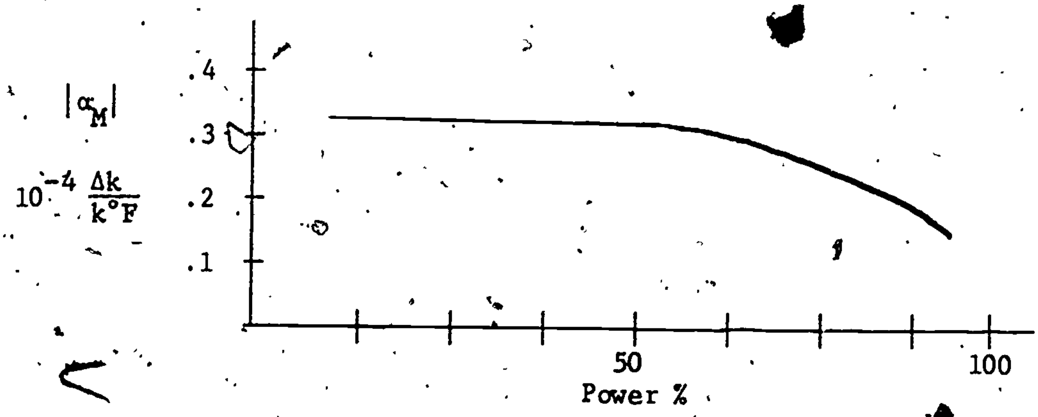
2.3 The Feedback Parameters

There are various parameters that must be obtained in order to numerically solve Equation (2.2.19). We list these parameters in order:

1. α_M - The moderator (coolant) temperature coefficient of reactivity.
2. α_F - The temperature coefficient of reactivity for the fuel.
3. C_{PF} - The total heat capacity of the fuel.
4. k_F - The thermal conductivity of the fuel.
5. h_T - The overall convective heat transfer coefficient of the fuel to moderator surface.

The flow rate F, the fuel volume V_F, etc. are all parameters that can be obtained from specifications of a particular reactor type and we shall not discuss these any further.

We first obtain a relation for α_M. The Final Safety Analysis Report of nuclear power plants generally have a curve of α_M as a function of power level for a given boron insertion and a critical rod insertion. The curves generally follow the pattern that α_M is constant from about 10% to 60% power and then decreases rather dramatically between 60 and 100% power. α_M is assumed to be negative.



We will therefore take α_M to be an input constant but an accurate analysis would necessitate a knowledge of the variation of α_M with power.

The $\alpha_F(P)$ is also rather difficult to calculate. To do this, we use Equation (2.2.7), i.e.

$$\alpha_F \approx \frac{1}{k_{eff}} \frac{\partial k_{eff}}{\partial T_F} . \tag{2.3.1}$$

Now if we use the relation

$$k_{eff} = \eta p\ f \varepsilon\ L_f\ L_{th} \tag{2.3.2}$$

then

$$\ell n(k_{eff}) = \ell n(\eta\ f\varepsilon\ L_f\ L_{th}) + \ell n(p)$$

and if we assume that the resonance escape probability p is the only factor which changes with the fuel temperature, then

$$\alpha_F = \frac{1}{k_{eff}} \frac{dk_{eff}}{dT_F} = \frac{1}{p} \frac{dp}{dT_F} . \tag{2.3.3}$$

A standard expression for the resonance escape probability is (2)

$$p = e^{-\frac{N_F V_F}{\xi_M \Sigma_{SM} V_M} R(T_F)} \tag{2.3.4}$$

and the resonance integral $R(T_F)$ is given by the empirical relation

$$R(T_F) = R(T_{FO})[1 + \gamma(\sqrt{T_F} - \sqrt{T_{FO}})] \tag{2.3.5}$$

where T_{FO} is the equilibrium temperature of the fuel.

We now relate "p" to α_F in a way so that the α_F can be calculated. Let

$$a_5 \equiv \frac{N_F V_F}{\xi_M \Sigma_{SM} V_M} \tag{2.3.6}$$

then

$$\ell n\ p = -\ a_5\ \overline{R}(T_F), \tag{2.3.7}$$

and

$$\alpha_F = -\ a_5\ \frac{dR(T_F)}{dT_F} = -\ a_5\ \frac{R(T_{FO})\gamma}{2\sqrt{T_F}}. \tag{2.3.8}$$

IF we substitute T_{FO} into p we get

$$p(T_{FO}) = e^{-a_5\ R(T_{FO})}$$

or

$$\ell n\ \frac{1}{p(T_{FO})} = a_5\ R(T_{FO}). \tag{2.3.9}$$

For an arbitrary value of temperature T_F we obtain

$$\ell n\ \frac{1}{p(T_F)} = a_5\ R(T_F). \tag{2.3.10}$$

Dividing Equation (2.3.9) by Equation (2.3.10) we get

$$\frac{R(T_{FO})}{R(T_F)} = \frac{\ell n\ \frac{1}{p(T_{FO})}}{\ell n\ \frac{1}{p(T_F)}} = +\ \frac{\ell n\ p(T_{FO})}{\ell n\ p(T_F)} \tag{2.3.11}$$

Inserting Equation (2.3.11) into Equation (2.3.8) we obtain

$$\alpha_F = -\frac{\gamma}{2\sqrt{T_F}}\left[a_5 R(T_{FO})\right] = -\frac{\gamma a_5}{2\sqrt{T_F}} R(T_F) \cdot \frac{\ell n\ p\ (T_{FO})}{\ell n\ p\ (T_F)} .$$

Equation (2.3.10) yields

$$a_5 R(T_F) = -\ell n\ p(T_F)$$

so

$$\alpha_F = -\frac{\gamma}{2\sqrt{T_F}} \cdot \left[-\ell n\ p(T_F)\right] \cdot \frac{\ell n\ p(T_{FO})}{\ell n\ p(T_F)}$$

$$= -\frac{\gamma}{2\sqrt{T_F}} \frac{1}{\ell n\ p(T_{FO})} . \qquad (2.3.12)$$

In our module, we will assume that γ and $p(T_{FO})$ are read in.

The third item of discussion is C_{PF}, the total heat capacity of the fuel. It is obvious that

$$C_{PF} = V_F \cdot C'_{PF} \cdot \rho_F \qquad (2.3.13)$$

where C'_{PF} is the specific heat capacity of the fuel, i.e. the units are Btu/lbm-°F, ρ_F is the density of fuel and V_F is the volume of fuel. There is a small variation of C_{PF} with temperature but it is small for UO_2 in the region of interest. The value that will be of interest for us is 0.0590 Btu/lbm-°F.

The thermal conductivity of the fuel is a function of the temperature. El Wakil (3) lists values of k_F as a function of T_F. We use these values to obtain a polynomial regression of the k_F with the T_F.

The last quantity that we discuss is the overall convective heat transfer coefficient h_T. The heat transfer coefficient is defined by Newton's law of cooling. The relevant relation is

$$\frac{P(t)}{A_F} = h_T(T_F - T_M), \qquad (2.3.14)$$

where A_F is the fuel area. There are many factors which influence h_T such as

i) the temperature of the system

ii) the heat flux

iii) the physical properties of the moderating material

iv) the geometrical shape of the cooling surface

v) the flow rate of the coolant

In the PWR, the coolant flow is turbulent. Therefore, to obtain the heat transfer coefficient h_T we assume that

$$h_T = \frac{Ck_M Re^{0.8} Pr^{0.333}}{D_e} \qquad (2.3.15)$$

where

D_e = The equivalent diameter of flow channels through the fuel rod bundles

k_M = The thermal conductivity of the moderator

C = The Colburn correction factor for fluid flow parallel to the tube bundles

Re = Reynolds number

Pr = Prandtl number of the coolant in the core

Equation (2.3.15) was a correlation recommended by Weisman. The Prandtl-number is taken from the 1967 ASME Steam Tables.

The equivalent diameter of the flow channel is shown in Fig. 2.3.2. If we let "a" be the area of the flow channel and P_w be the wetted perimeter, then

$$D_e = \frac{4A}{P_w} \tag{2.3.16}$$

For a typical PWR, D, the diameter of the fuel rods is 0.03583 ft and the pitch (the distance between centerlines of fuel rods) is designated as P_c. The Colburn correction factor is an empirical relation and is

$$C = 0.042 \frac{P_c}{D} - 0.024. \tag{2.3.17}$$

So we denote the heat transfer coefficient, which varies with power and therefore with time, as $h_T(t)$ and the equilibrium value as h(0). Using Equation (2.3.15) we have

$$\frac{h_T(t)}{h_T(0)} = \frac{k_M(t)\ Re^{0.8} Pr^{1/3}}{k_M(0)\ Re^{0.8}(0)\ Pr^{1/3}(0)},$$

or

$$h_T(t) = \frac{h(0)\ k_M(t)}{k_M(0)\ Pr^{1/3}(0)} \frac{\rho_\mu(t)}{\mu(t)}^{0.8} \frac{Pr(t)^{1/3}}{\left(\frac{\rho_M(0)}{\mu\ (0)}\right)^{0.8}}, \tag{2.3.18}$$

where we have used the fact that

$$Re = \frac{\rho_M vD_e}{\mu}, \tag{2.3.19}$$

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$$\mu_M(T) = 8.5067\text{x}10^{-1} - 1.7501\text{x}10^{-3}\ T_M + 1.1142\text{x}10^{-6}\ T_M^2 \qquad (2.3.23)$$

and for the thermal conductivity of the fuel, we have:

$$k_F(T) = 6.102141 - 4.636522\text{x}10^{-3}\ T_F + 1.306299\text{x}10^{-6}\ T_F^2. \qquad (2.3.24)$$

The temperature is for the fuel in the case of k_F but the moderator temperature otherwise.

Problem 2.3.1

Given the following data:

R_F = 0.015042 ft

Fuel length = 12.00 ft

Volume of reactor vessel = 3643 ft^3

Number of fuel assemblies = 145

Fuel rods per assembly = 208

Rod pitch = 0.04733 ft

ρ = 43.214 lbm/ft^3

v = 16.3 ft/sec

μ = 5.828×10^{-5}

D_e = 0.04377 ft

Re = 528,998

Pr = 1.01

k_M = 0.3010 Btu/hr-ft-°F

a) show the Colburn correction is 0.03148

b) h = 8232 Btu/ft^2-hr-°F.

2.4 Numerical Solution of the Dynamics Equations

The numerical integration of the differential equations can be carried out once the various parameters, such as the heat transfer coefficient, thermal conductivity, etc., have been obtained. We again write the equations so that we can proceed to use Hansen's method in their solution. FUMOTEM solves the kinetics equations using Hansen's method. The system of equations is

$$\frac{d\underline{\phi}(t)}{dt} = \underline{\underline{A}}\,\underline{\phi}(t) + \underline{B} \qquad (2.4.1)$$

where

$$\underline{\phi} = \begin{bmatrix} P(t) \\ Q(t) \\ T_F(t) \\ T_M(t) \end{bmatrix} \qquad \underline{B} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \dfrac{2\,\dot{m}_M T_{MI}(t)}{\rho_M V_M} \end{bmatrix} \qquad (2.4.2)$$

and

$$\underline{\underline{A}} = \begin{bmatrix} \dfrac{\rho(t)-\beta}{\Lambda} & \lambda & 0 & 0 \\ \dfrac{\beta}{\Lambda} & -\lambda & 0 & 0 \\ \dfrac{1}{2\rho_F C_F V_F} & 0 & -\dfrac{4\pi k_F L_F N}{\rho_F C_F V_F} & \dfrac{4\pi k_F L_F N}{\rho_F C_F V_F} \\ \dfrac{1}{\rho_M C_M V_M} & 0 & 0 & \dfrac{-2\,\dot{m}_M}{\rho_M V_M} \end{bmatrix} \qquad (2.4.3)$$

subject to the constraint that

$$\rho(t) = \rho_o + \alpha_M \Delta T_M + \alpha_F \Delta T_F. \tag{2.4.4}$$

All the parameters involved in Equations (2.4.1) - (2.4.4) are defined in section 2.2.

Hansen's method must be generalized slightly for the solution of Equation (2.4.1). We break the $\underline{\underline{A}}$ matrix into three parts:

$$\underline{\underline{A}} = \underline{\underline{L}} + \underline{\underline{D}} + \underline{\underline{U}}, \tag{2.4.5}$$

with

$$\underline{\underline{D}}(t) = \begin{bmatrix} \frac{\rho(t)-\beta}{\Lambda} & 0 & 0 & 0 \\ 0 & -\lambda & 0 & 0 \\ 0 & 0 & \frac{-4\pi k_F L_F' N}{\rho_F C_F V_F} & 0 \\ 0 & 0 & 0 & \frac{-2\dot{m}_M}{\rho_M V_M} \end{bmatrix}$$

$$L = \begin{bmatrix} 0 & 0 & 0 & 0 \\ \frac{\beta}{\Lambda} & 0 & 0 & 0 \\ \frac{1}{2\rho_F C_F V_F} & 0 & 0 & 0 \\ \frac{1}{\rho_M C_M V_M} & 0 & 0 & 0 \end{bmatrix}$$

and

$$\underline{\underline{U}} = \begin{bmatrix} 0 & \lambda & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \dfrac{4\pi k_F L_F N}{\rho_F C_F V_F} \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Equation (2.4.1) can then be written as

$$\frac{d\underline{\phi}}{dt} - \underline{\underline{D}}(t)\,\underline{\phi}(t) = (\underline{\underline{L}} + \underline{\underline{U}})\,\underline{\phi}(t) + \underline{B} . \tag{2.4.6}$$

We wish to develop an iteration procedure for the solution of this system so we begin the calculation at time t_0 and advance to time t_1, and define

$$h = t_1 - t_0 . \tag{2.4.7}$$

Before we use this relation though, let's multiply each term of Equation (2.4.6) by the integrating factor $e^{-\int_0^t \underline{\underline{D}}(t')dt'}$ so

$$e^{-\int_0^t \underline{\underline{D}}(t')dt'} \frac{d\underline{\phi}}{dt} - e^{-\int_0^t \underline{\underline{D}}(t')dt'} \underline{\underline{D}}(t)\underline{\phi}(t) = e^{-\int_0^t \underline{\underline{D}}(t')dt'} (\underline{\underline{L}}+\underline{\underline{U}})\phi(t) + e^{-\int_0^t \underline{\underline{D}}(t')dt'} \underline{B}$$

or

$$\frac{d}{dt}\{e^{-\int_0^t \underline{\underline{D}}(t')dt'} \underline{\phi}(t)\} = e^{-\int_0^t \underline{\underline{D}}(t')dt'} (\underline{\underline{L}} + \underline{\underline{U}})\phi(t) + e^{-\int_0^t \underline{\underline{D}}(t')dt'} \underline{B}. \tag{2.4.8}$$

The left side is an exact differential so we integrate from 0 to h and obtain

$$e^{-\int_0^t \underline{\underline{D}}(t')dt'} \underline{\phi}(t) \Big|_0^h = \int_0^h e^{-\int_0^t \underline{\underline{D}}(t')dt'} (\underline{\underline{L}}+\underline{\underline{U}})\underline{\phi}(t)dt + \int_0^h e^{-\int_0^t \underline{\underline{D}}(t')dt'} \underline{B}(t)dt$$

or

$$\underline{\phi}(t_o + h) = e^{+\int_o^h \underline{\underline{D}}(t')dt'} \underline{\phi}(t_o) + \int_o^h e^{\int_o^h \underline{\underline{D}}(t')dt'} e^{-\int_o^t \underline{\underline{D}}(t')dt'} (\underline{\underline{L}} + \underline{\underline{U}})\underline{\phi}(t_o+\Theta)d\Theta$$

$$+\int_o^h e^{+\int_o^h \underline{\underline{D}}(t')dt' - \int_o^t \underline{\underline{D}}(t')dt'} \underline{B}(t_o + \Theta)d\Theta . \qquad (2.4.9)$$

Here

$$t_o < \Theta < t_1 = t_o + h$$

and

$$d\Theta = dt.$$

Notice in Equation (2.4.9) that

$$e^{\int_o^h \underline{\underline{D}}(t')dt' - \int_o^t \underline{\underline{D}}(t')dt'} = e^{\int_t^h \underline{\underline{D}}(t')dt'} = e^{\int_{t_o+\Theta}^h \underline{\underline{D}}(t')dt'}$$

Therefore the equation that must be solved is

$$\underline{\phi}(t_o + h) = e^{\int_o^h \underline{\underline{D}}(t')dt'} \phi(t_o) + \int_o^h e^{\int_{t_o+\Theta}^h \underline{\underline{D}}(t')dt'} (\underline{\underline{L}} + \underline{\underline{U}})\Big|_{t_o+\Theta} \underline{\phi}(t_o + \Theta)dt$$

$$+\int_0^h e^{\int_{t_0+\Theta}^h \underline{\underline{D}}(t')dt'} \underline{B}(t_0 + \Theta)d\Theta \quad . \tag{2.4.10}$$

Equation (2.4.10) is an integral equation so we must approximate its solution. To do this, we assume that

$$\underline{\phi}(t_0 + \Theta) = e^{\omega_0\Theta}\,\underline{\phi}(t_0) \tag{2.4.11}$$

where ω_0 is the largest eigenvalue of the matrix $\underline{\underline{A}}$ evaluated at time t_0. This means we must solve the equation

$$\left|\underline{\underline{A}} - \omega\underline{\underline{I}}\right| = 0$$

or

$$\det\begin{bmatrix} \frac{\rho(t_0)-\beta}{\Lambda} - \omega & \lambda & 0 & 0 \\ \frac{\beta}{\Lambda} & -\lambda-\omega & 0 & 0 \\ \frac{1}{2\rho_F C_F V} & 0 & \frac{-4\pi k_F L_F N}{\rho_F C_F V_F} - \omega & \frac{4\pi k_F L_F N}{\rho_F C_F V_F} \\ \frac{1}{\rho_M C_M V_M} & 0 & 0 & \frac{-2\dot{m}_M}{\rho_M V_M} - \omega \end{bmatrix} = 0, \tag{2.4.12}$$

to obtain the eigenvalues, ω_0, ω_1, ω_2 and ω_3, where

$$\omega_0 > \omega_1 > \omega_2 > \omega_3 \; .$$

The ω's are, of course, time dependent quantities.

We also look at $\underline{B}(t_o + \Theta)$ and assume that we can expand it in a Taylor series so that

$$\underline{B}(t_o + \Theta) = \underline{B}(t_o) + \Theta \frac{d\underline{B}}{dt}(t_o) + \frac{1}{2}\Theta^2 \frac{d^2\underline{B}(t_o)}{d\Theta^2} + \ldots \qquad (2.4.13)$$

and also

$$\underline{\underline{L}}(t_o + \Theta) + \underline{\underline{U}}(t_o + \Theta) \simeq \underline{\underline{L}}(t_o) + \underline{\underline{U}}(t_o) + \ldots \qquad (2.4.14)$$

We shall keep only the 1st terms of these expansions.

Inserting Equations (2.4.14), (2.4.13) and (2.4.11) into Equation (2.4.10) we get (after assuming that $\int_0^h \underline{\underline{D}}(t')dt' \simeq \underline{\underline{D}}(t_o)h$)

$$\underline{\phi}(t_o + h) = e^{\underline{\underline{D}}h}\underline{\phi}(t_o) + \int_0^h e^{\underline{\underline{D}}(h-\Theta)} (\underline{\underline{L}} + \underline{\underline{U}})\, e^{\omega_o \Theta}\, \underline{\phi}(t_o)\, d\Theta$$

$$+ \int_0^h e^{\underline{\underline{D}}(h-\Theta)}\, \underline{B}(t_o)\, d\Theta$$

$$= e^{\underline{\underline{D}}h}\underline{\phi}(t_o) + e^{\underline{\underline{D}}h}(\omega_o\underline{\underline{I}} - \underline{\underline{D}})^{-1} \left[e^{(\omega_o\underline{\underline{I}} - \underline{\underline{D}})\Theta} \right]_0^h (\underline{\underline{L}} + \underline{\underline{U}})\underline{\phi}(t_o)$$

$$+ e^{\underline{\underline{D}}h}(-\underline{\underline{D}}^{-1})\, e^{-\underline{\underline{D}}\Theta} \Big|_0^h \underline{B}(t_o).$$

Putting the limits into this equation and simplifying we finally have

$$\underline{\phi}(t_o + h) = e^{\underline{\underline{D}}h}\,\underline{\phi}(t_o) + (\omega_o\underline{\underline{I}} - \underline{\underline{D}})^{-1}[e^{\omega_o h\underline{\underline{I}}} - e^{\underline{\underline{D}}h}][\underline{\underline{L}}(t_o) + \underline{\underline{U}}(t_o)]\underline{\phi}(t_o)$$

$$+ \underline{\underline{D}}^{-1}(e^{\underline{\underline{D}}h} - \underline{\underline{I}})\underline{B}(t_o) \quad . \qquad (2.4.15)$$

If we now set

$$\underline{\phi}(t_j) = \underline{\phi}_j$$

and

$$\underline{\phi}(t_j + h) = \underline{\phi}_{j+1} \, .$$

Then Equation (2.4.15) becomes

$$\underline{\phi}_{j+1} = \underline{\underline{H}}\,\underline{\phi}_j + \underline{\underline{R}}\,\underline{B}_j \qquad (2.4.16)$$

where

$$\underline{\underline{H}} \equiv e^{\underline{\underline{D}}h} + (\omega_o\underline{\underline{I}} - \underline{\underline{D}})^{-1}[e^{\omega_o\underline{\underline{I}}h} - e^{\underline{\underline{D}}h}](\underline{\underline{L}} + \underline{\underline{U}}) \qquad (2.4.17)$$

with all quantities evaluated at time t_o and

$$\underline{\underline{R}} = \underline{\underline{D}}^{-1}(e^{\underline{\underline{D}}h} - \underline{\underline{I}}) \, . \qquad (2.4.18)$$

We now write $\underline{\underline{H}}$ and $\underline{\underline{R}}$ explicitly for our problem. To do this, consider Equation (2.4.17) and the explicit relation for $\underline{\underline{L}}$ and $\underline{\underline{U}}$; then

$$\underline{\underline{H}} = \begin{bmatrix} H_1 & H_2 & 0 & 0 \\ H_3 & H_4 & 0 & 0 \\ H_5 & 0 & H_6 & H_7 \\ H_8 & 0 & 0 & H_9 \end{bmatrix}, \qquad (2.4.19)$$

where

$$H_1 = e^{\left[\frac{\rho(t_o)-\beta}{\Lambda}\right]h} ,$$

$$H_2 = \lambda\left[\frac{e^{\omega_o h} - e^{\frac{\rho(t_o)-\beta}{\Lambda}h}}{\omega_o - \frac{\rho(t_o)-\beta}{\Lambda}}\right] ,$$

$$H_3 = \frac{\beta}{\Lambda}\left[\frac{e^{\omega_o h} - e^{-\lambda h}}{\omega_o + \lambda}\right] ,$$

$$H_4 = e^{-\lambda h} ,$$

$$H_5 = \frac{1}{2\rho_F C_F V_F}\left[\frac{e^{\omega_o h} - e^{\frac{4\pi k_F L_F N}{\rho_F C_F V_F}h}}{\omega_o - \frac{4\pi k_F L_F N}{\rho_F C_F V_F}}\right],$$

$$H_6 = e^{-\frac{4\pi k_F L_F N}{\rho_F C_F V_F}h} ,$$

$$H_7 = \frac{4\pi k_F L_F N}{\rho_F C_F V_F}\left[\frac{e^{\omega_o h} - e^{\frac{4\pi k_F L_F N}{\rho_F C_F V_F}h}}{\omega_o - \frac{4\pi k_F L_F N}{\rho_F C_F V_F}}\right] .$$

$$H_8 = \frac{1}{\rho_M C_M V_M}\left[\frac{e^{\omega_o h} - e^{-\frac{2\dot{m}_M h}{\rho_M V_M}}}{\omega_o + \frac{2\dot{m}_M}{\rho_M V_M}}\right],$$

and

$$H_9 = -\frac{2\dot{m}_M}{\rho_M V_M}\left[\frac{e^{\omega_o h} - e^{-\frac{2\dot{m}_M}{\rho_M V_M} h}}{\omega_o + \frac{2\dot{m}_M}{\rho_M V_M}}\right],$$

and the $\underline{\underline{R}}$ matrix is

$$\underline{\underline{R}} = \begin{bmatrix} \frac{\Lambda}{\rho-\beta}\left(e^{\frac{\rho-\beta}{\Lambda}}-1\right) & 0 & 0 & 0 \\ 0 & \frac{1}{\lambda}(1-e^{-\lambda h}) & 0 & 0 \\ 0 & 0 & \frac{\rho_F C_F V_F}{4\pi k_F L_F N}\left(1-e^{-\frac{4\pi k_F L_F N h}{\rho_F C_F V_F}}\right) & 0 \\ 0 & 0 & 0 & \frac{\rho_M V_M}{2\dot{m}_M}\left(1-e^{\frac{-2\dot{m}_M}{\rho_M V_M} h}\right) \end{bmatrix} \quad (2.4.20)$$

The basic iteration procedure in FUMOTEM is as follows:

1. Calculate the pertinent reactor parameters such as α_F, h_T, $\dot{m}$, ρ_M, μ, C_M, k_F, and k_M for the guessed initial conditions. The parameters read in are T_{M1}, α_M, h, P_o, v (the coolant velocity), some optional settings for print out and the time length the program is to operate.

2. Construct the vector $\underline{\phi}(0)$. We can choose the parameters of interest, i.e.

$$P(0) = 500 \text{ MW},$$

and

$$Q(0) = \frac{\beta}{\Lambda\lambda} P(0) \text{ MW}.$$

The fuel temperature $T_F(0)$ and the moderator temperature $T_M(0)$ are completely determined by $P(0)$ and the above read-in parameters. $T_F(0)$ and $T_M(0)$ are difficult to obtain since C_M, k_M, μ etc. depend on the temperature. This difficulty is overcome by using an iteration procedure. The method is as follows:

a) From $P(0)$ and initially guessed values $T_F^{(0)}(0)$ and $T_M^{(0)}(0)$, calculate $\dot{m}_M$ from the relation

$$\dot{m}_M(T_M) = \rho_M(T_M)\ Av. \tag{2.4.21}$$

Equation (2.3.20) is used to obtain $\rho_M(T_M)$ from the guessed moderator temperature $T_M^{(0)}(0)$. Equation (2.2.15) is now used to obtain an improved guessed power $P^{(1)}(0)$,

$$P^{(1)}(0) = 2\dot{m}_M C_M (T_M^{(0)}(0) - T_{M1}). \tag{2.4.22}$$

b) The fractional difference between the actual power and $P^{(1)}(0)$ is

$$E \equiv \frac{|P^{(1)}(0) - P(0)|}{P(0)} \tag{2.4.23}$$

If $E \leq \Delta$ ($\Delta = 0.01$ and is an input) then the guessed temperature $T_M^{(0)}(0)$ is the correct moderator temperature. If

$$E > \Delta \tag{2.4.24}$$

then change $T_M^{(0)}(0)$ to $T_M^{(1)}(0)$ and repeat the above until convergence is achieved.

c) The $T_M(0)$ is now used in Equation (2.2.17) which, for equilibrium, becomes, after solving for $T_F(0)$,

$$T_F(0) = T_M(0) + \frac{1}{8\pi k_F L_F N} P(0). \tag{2.4.25}$$

This step also requires an iterative procedure since k_F is a function of the fuel temperature as seen from Equation (2.4.25).

3. Determine the largest eigenvalue of the equation

$$|\underline{\underline{A}} - \omega \underline{\underline{I}}| = 0.$$

This will be the solution of a 4 x 4 determinant which is rather easy on the computer. From this determine the largest root ω_o. The Newton-Raphson method is used to calculate all four roots of this polynomial and then the largest root is picked by comparison of the roots. The subroutine POLRT is used for the determination of ω_o for each time step. Module RD-1 describes the Newton-Raphson method.

4. Construct the $\underline{\underline{H}}(t_o)$ matrix using Equation (2.4.19) and $\underline{\underline{R}}(t_o)$ using (2.4.20)

5. Determine the vector $\underline{\phi}_1$ from Equation (2.4.16), i.e.

$$\underline{\phi}_1 = \underline{\underline{H}}(t_o)\underline{\phi}_o + \underline{\underline{R}}(t_o)\underline{B}_o.$$

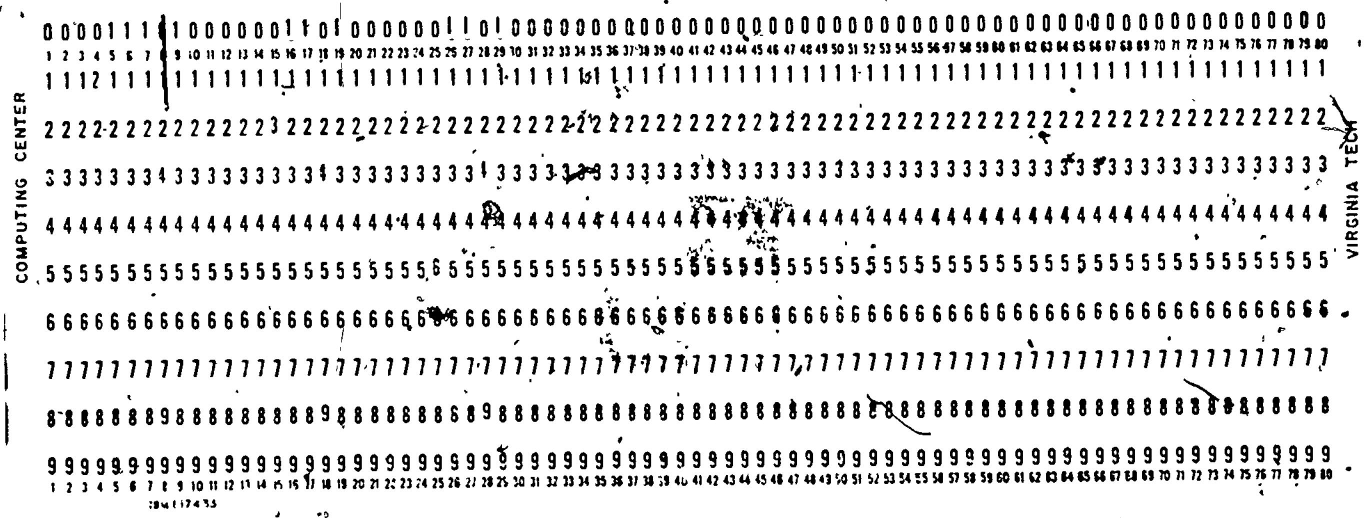
6. Repeat the above procedure using step 3 and continue as long as required to achieve the solution over the time domain of interest. The time steps are chosen by the criterion $h = \left|\frac{1}{\omega_o}\right|$ and h is constrained to be in the interval $0.005 \le h \le 0.05$ sec.

2.5 Input-Output Data for Code FUMOTEM

The input data required for the program are presented below:

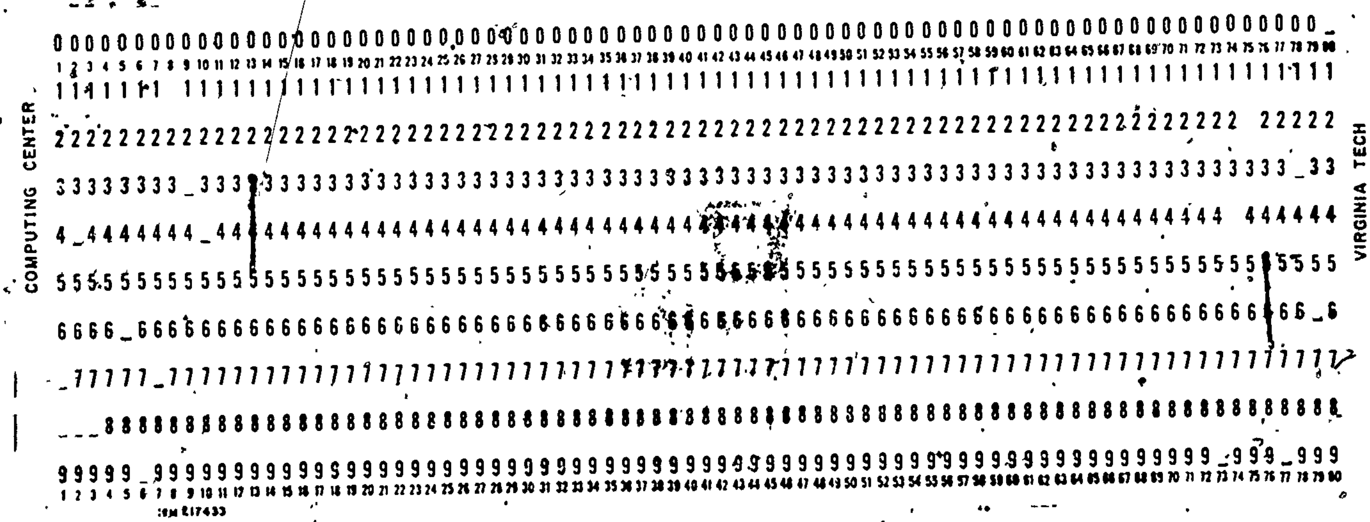
Data Card Number	Format Statement Number	Format	Unit	Variable Name	Description
1	20	I2	--	NOPLT	Plotting option 1 -- plot 0 -- no plot
2	30	F10.5	--	BETA	Delayed neutron fraction
		F10.5	sec^{-1}	X	Delayed neutron decay constant
		F10.7	sec^{-1}	XL	Neutron generation time
		F10.7	\$	RO	reactivity, ρ_o
		I10	--	NRO	option for the type of reactivity insertion
		F10.5	sec	RTIME	Time duration reactivity is inserted for ramp input (NRO=2)
		F10.5	sec^{-1}	A	reactivity insertion rate
3	40	F10.6	$°F^{-1}$	AM	Moderator temperature coefficient of reactivity
		F10.6	$(°F)^{1/2}$	G	Constant for resonance capture
		F10.6	--	PTFO	Resonance escape probability, P_o
4	50	F10.5	ft	RF	Fuel radius, R_F
		F10.5	ft	PC	Distance between fuel pin centers, P
		F10.5	$\frac{Btu}{lb-°F}$	CPFX	Specific heat of the fuel, C_F
		F10.2	lb/ft^3	FDENS	Fuel density, ρ_F
		F10.2	ft	FH	Fuel height, L
		I10	--	NA	Number of fuel assemblies
		I10	--	NFRPA	Number of fuel rods per assembly
5	60	F10.2	ft/sec	V	Average velocity of coolant, v
		F10.2	°F	TMI	Inlet coolant temperature to the core T_{M1}.
6	70	F10.4	sec	TE	End of calculation time
		F10.4	sec	DH	Time increment
		F10.4	--	DELTA	Convergence criterion for the calculation of the initial equilibrium state
7	80	F10.2	MW	PPW	Initial equilibrium power
		F10.2	°F	TGUES	Guessed initial moderator temperature to obtain equilibrium
		F10.2	°F	TGF	Guessed initial fuel temperature for obtaining equlibrum conditions.

1								①
0.00645	0.07695	0.0001	+0.50		2	0.5	2.5	②
-0.0001	0.00002	0.80						③
0.01504	0.04733	0.0590	43.2	12.0		145	208	④
13.0	400.0							⑤
8.0	0.01	0.01						⑥
1000.0	200.0	500.0						⑦



In addition to these seven data cards, there is an eighth card which has the impression as shown below. This card is necessary for the plot routine.

f*Q FRP1LM RD2 9361



The output of FUMOTEM consists of three parts. The first is simply a writing of all the input data. The second is the equilibrium state calculation and the third block of data is the fuel temperature, moderator temperature reactivity, exit temperature, power, α_F and Q as a function of time. If the user specifies, a plot of power, precursor power, T_F, T_M and reactivity as a function of time is provided.

Problem 2.5.1

Run FUMOTEM for the sample data cards shown.

Problem 2.5.2

Run FUMOTEM for the reactivity input

$$\rho(t) = \$0.50 \cos 2.5t$$

with the following parameters:

NOPLT = 1	$\alpha_M = -0.00005$
$\beta = 0.00645$	
$\lambda = 0.07695\ \text{sec}^{-1}$	$\gamma = 0.0002$
RTIME = 4.0 sec	$P_o = 0.80$
$R_F = 0.01504$ ft	Number of fuel assemblies = 745
$P_C = 0.04733$ ft	
$C_{PF} = 0.0590$ Btu/lb-°F	Number of fuel pins per assembly = 208
$\rho_F = 43.2\ \text{lb/ft}^3$	
L = 12 ft	$T_{M1} = 400°F$
TE = 4.0 sec	DELTA = 0.01

DH = 0.01 sec

P_o = 1000 MW

TGUES = 200.0°F

TGF = 500.0°F

v = 13.0 ft/sec

Also run the same calculation for v = 26.0 ft/sec.

FUMOTEM is written in FORTRAN in single precision except for the eigenvalue calculation. The solution of the equation

$$|\underline{\underline{A}} - \omega\, \underline{\underline{I}}| = 0$$

for the root ω_0, ω_1, ω_2, ω_3 is done in double precision. All four roots are determined and the largest one is picked to form the $\underline{\underline{H}}$ and $\underline{\underline{R}}$ matrices.

The memory requirement is about 40 kilobytes and the execution time varies with the time length. The reactor is to be simulated. Generally it takes about 1 1/2 seconds of computer time to simulate one second of reactor transient time.

REFERENCES

1. D. L. Hetrick, "Dynamics of Nuclear Reactors," The University of Chicago Press, 1971, page 159.

2. J. R. Lamarsh, "Nuclear Reactor Theory," Addison-Wesley Publishing Company, Reading, Massachusetts, 1966, pages 218, 459.

3. M. M. El-Wakil, "Nuclear Heat Transport," International Textbook Company, Scranton, Pa., 1971, page 104.

List of Symbols for FUMOTEM

The following symbols are listed in alphabetical order in the FUMOTEM program.

A	a	Period of reactivity insertion
AF	α_F	Fuel temperature coefficient
AM	α_M	Moderator temperature coefficient
AMTRX		Subroutine to form the $\underline{\underline{A}}$ matrix
AMX		Elements of the $\underline{\underline{A}}$ matrix
AX		Coefficients of the EIGEN4 polynomial, i.e., if $\sum_{i=1}^{5} a_n\omega^n$, the a_n.
BE		$\underline{B}$ matrix element.
BETA	β	Delayed neutron fraction
CK		Function subroutine to calculate the water thermal conductivity as a function of temperature (Equation 2.3.21)
COL	C	Colburn number (Equation 2.3.17)
CP	C_{PM}	Heat capacity of moderator as a function of T_M
CPF	NC_{PF}	Total heat capacity of the fuel
CPFX	C_{PF}	Heat capacity of a single fuel rod
CPG, CPMX	C_{PM}	Total heat capacity of the water
CX, CGUES	k_M	Thermal conductivity of the water
DE	De	Equivalent diameter (Equation 2.3.16)
DELTA	Δ	Convergence criterion for equilibrium calculation

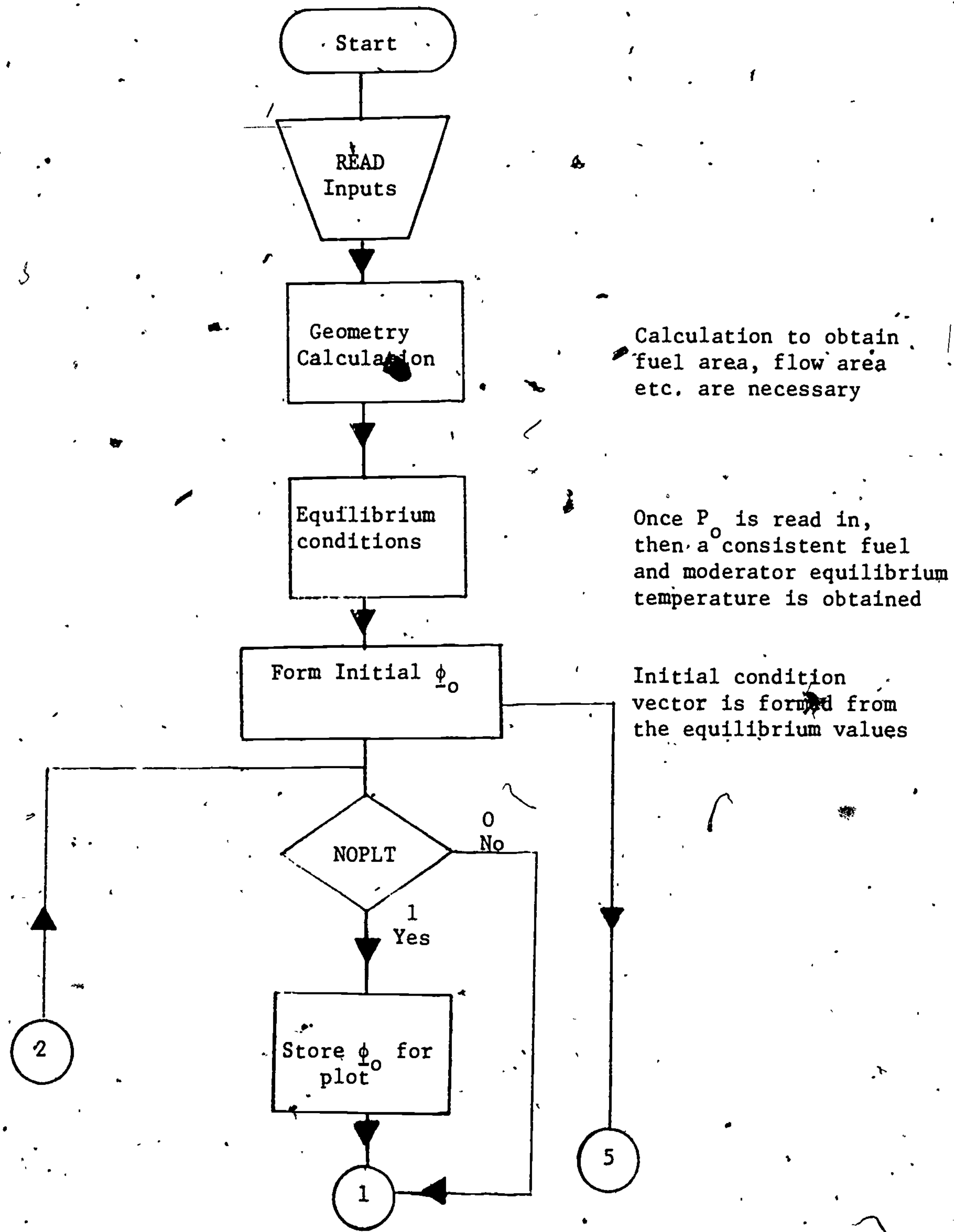
DF	$2R_F$	Fuel diameter
DH	Δt	Time increment (TINCR)
DH1		Lower limit time increment
DH2		Upper limit time increment
DM	t	Time at any instant
DOTMG	$\dot{m}_M$	Mass flow of the coolant
DX, DGUES	ρ_M	Density of the coolant
EIGEN4		Subroutine to change $\lvert \underline{\underline{A}} - \omega \underline{\underline{I}} \rvert = 0$ to polynomial form
EIGENV	ω	Eigenvalues of the $\underline{\underline{A}}$ matrix
FA		Heat transfer area
FCA		Fuel cross section area
FDENS	ρ_F	Fuel density
FH	L	Fuel rod length
FK		Function subroutine used to calculate the fuel conductivity as a function of temperature
FLA		Flow area
FX, FGUES	k_F	Thermal conductivity of fuel
G	γ	Constant in Equation (2.3.12)
GXN		Subroutine to multiply an nxn matrix with a column matrix
H	h_T	Heat transfer coefficient
HH		Elements of the $\underline{\underline{H}}$ matrix (Equation 2.4.19)
HMTRX	$\underline{\underline{H}}$	Subroutine to form the $\underline{\underline{H}}$ matrix
HP		Element of the column matrix $\underline{\underline{H}}\underline{\phi}$
HX, HGUES	h_T	Heat transfer coefficient

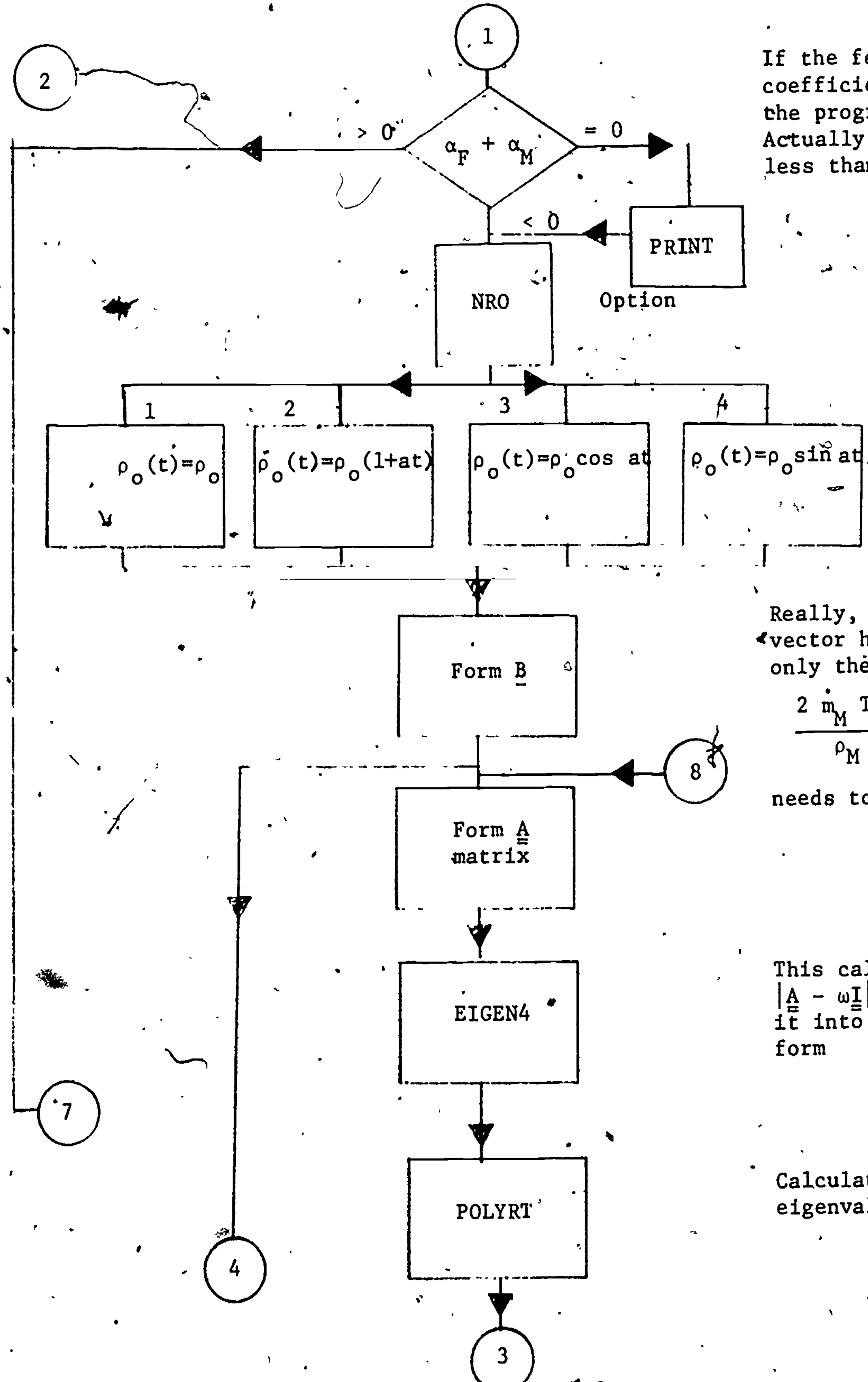
MTRX		Order of the nxn matrix
MX	t	Dimensioned time
N		Number of iterations
NA		Number of fuel assemblies
NCHAN		Total number of coolant channels
NFRPA		Number of fuel rods per assembly
NIN		Number of iterations calculated from insertion time and initial time increment
NN		Total number of iterations, calculated from end time and initial time increment
NOPLT		Option for plotting
NR		READ statement unit number
NRO		Option for reactivity insertion
NW		WRITE statement unit number
NZ		Dummy variable used for print out
PAREA		Square pitch area for channel
PC	P_C	Distance between fuel pins
PI	π	3.14159
PKL		$4\pi k_F L_F$
POW	P_o	Input power in MW
PP	P	Power in Btu/hr
PR	Pr	Prandtl number function subroutine
PW	P_w	Wetted perimeter
QQ, QW	Q	Precursor density (Power)
RIN		Reactivity inserted
RB		Element of column matrix $\underline{\underline{R}}\ \underline{B}$

RCOS	$\rho = \rho_o \cos a t$	Function subroutine to calculate reactivity for a cosine insertion
RE	Re	Function subroutine to calculate the Reynolds number
RF	R_F	Fuel radius
RHO	$\rho_M(T_M)$	Function subroutine to calculate density of H_2O
RL	$\rho = \rho_o(1+a t)$	Function subroutine to calculate reactivity for a ramp
RMTRX		Subroutine to form the $\underline{\underline{R}}$ matrix (Equation 2.4.20)
RN		Reynolds number
RO	ρ_o	Reactivity inserted at $t = 0$
RSIN	$\rho = \rho_o \sin a t$	Function subroutine to calculate reactivity for a sine insertion
RP	Pr	Prandtl number
RR	$\rho_T = \rho_{in} + \rho_f$	Total reactivity, inserted plus feedback
RTIME	t_r	Time when inserted reactivity is removed (for ramp reactivity only)
RX		Element of $\underline{\underline{R}}$ matrix (Equation 2.4.20)
RY		Dimensioned reactivity
TCFA		Total cross sectional area of the fuel
TCFLA		Total cross sectional area of coolant
TE		End of calculation time
TFA		Total heat transfer area
TFG		Initial guess for fuel temperature
TFO	T_{FO}	Equilibrium fuel temperature corresponding to P_o
TGUES, TMG		Initial guess for moderator temperature

TMI	T_{MI}	Inlet coolant temperature
TMO	T_{MO}	Equilibrium coolant temperature corresponding to a given P_o
TMOUT		Exit temperature of core water
TPLOT		Subroutine to plot five variables at the same time with respect to the independent variable time
U	μ	Function subroutine to calculate the viscosity of water as a function of temperature
UGUES, UX		Viscosity of water
V	V	Mean velocity of coolant
VF	V_F	Fuel volume
VM	V_M	Coolant volume
WO	ω_o	The largest eigenvalue to the equation $\lvert \underline{\underline{A}} - \omega \underline{\underline{I}} \rvert = 0$
X	λ	Decay constant of the delayed neutron group
XL	Λ	The neutron generation time

Flow Chart for FUMOTEM





If the feedback reactivity coefficients are positive, the program stops. Actually, $\alpha_F + \alpha_M$ must be less than or equal to zero.

One of these four reactivity insertions is to be used.

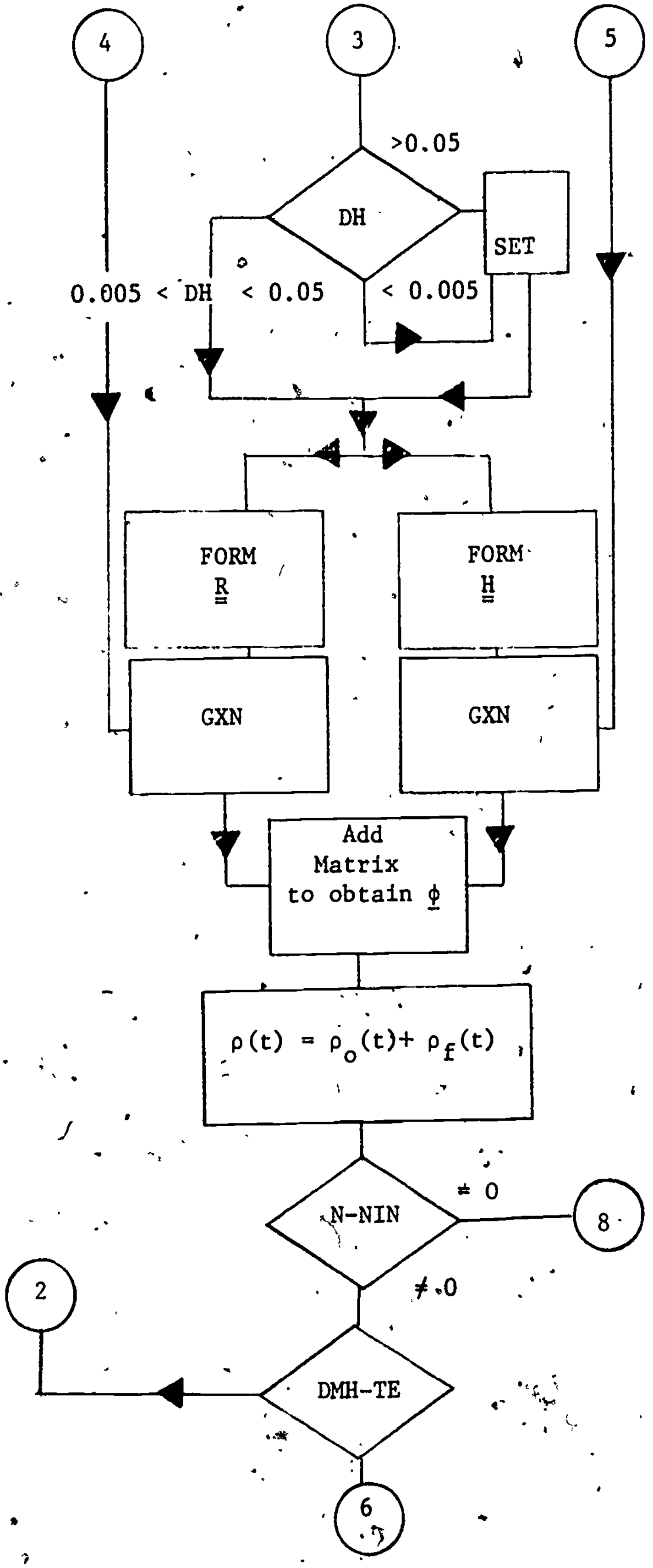
Really, since the $\underline{B}$ vector has 3 zeroes, only the number

$$\frac{2\,\dot{m}_M\,T_{M1}(t)}{\rho_M\,V_M}$$

needs to be solved.

This calculates the $|\underline{\underline{A}} - \omega\underline{\underline{I}}| = 0$ and puts it into polynomial form

Calculates the largest eigenvalue ω_o



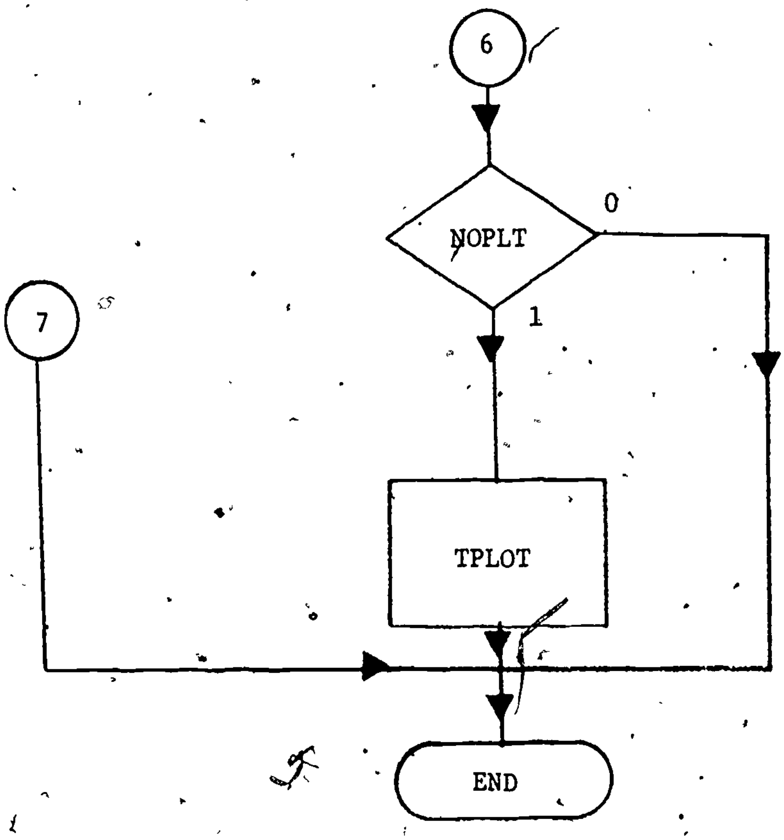
Δt is constrained to remain in the time interval $0.005 < \Delta t < 0.05$

GXN is a subroutine which multiplies matrices to obtain $\underline{\phi}$ (t)

Calculate total reactivity since know inserted and feedback components

If $t \geq t_r$, set $\rho(t) = \rho_f(t)$

If not at end of calculation, go back



This subroutine plots the state vector ϕ as a function of time.

```
//WATFIV       ONEGA,TIME=300,PAGES=50
C      -------------------------- PWR FEEDBACK -------------------------
C
C
C      CODE NAME     F U M O T E M .
C
C      OBJECTIVE
C              1.EXAMINE THE TEMPERATURE FEEDBACK MECHANISM OF A PWR AND
C
C              2.SOLVE THE ONE DELAYED NEUTRON MODEL WITH TEMPERATURE
C                FEEDBACK FOR A STEP INSERTION AND A RAMP INSERTION OF
C                REACTIVITY .
C
C      PROGRAM
C                WRITTEN IN SINGLE PRECISION .
C
C
C                    DESCRIPTIONS OF INPUT PARAMETERS .
C
C      FORMAT-  PARAMETER          FUNCTION                          UNIT
C      NUMBER
C
C      20       NOPLT       OPTION FOR PLOTTING
C                           1 - PLOT THE RESULT .
C                           0 - NO PLOT .
C
C      30       BETA        AVERAGE FRACTION OF DELAYED NEUTRONS ,
C                           IS 0.00645 , ONLY ONE DELAYED
C                           NEUTRON GROUP IS CONSIDERED .
C               X           DELAYED NEUTRON DECAY CONSTANT .        1/SEC .
C               XL          NEUTRON GENERATION TIME .                SEC
C               RO          INITIAL REACTIVITY INSERTED .
C                           INPUT RO IN DOLLAR UNIT .
C               NRO         OPTION FOR REACTIVITY INSERTION
C                           AS A FUNCTION OF TIME .
C                           1 - CONSTANT REACTIVITY .
C                           2 - LINEAR RAMP INSERTION .
C                           3 - COSINE VARIATION OF REACTIVITY .
C                           4 - SINE VARIATION OF REACTIVITY
C                               WITH TIME
C               RTIME       INSERTION TIME                          SEC
C               A           PERIOD , CONSTANT .                     1/SEC
C
C      40       AM          MODERATOR TEMPERATURE COEFFICIENT        1/F.
C               G           CONSTANT GAMMA IN EQUATION 2.3.13
C                           OF RD-2                                 SQRT(F)
C               PTFO        RESONANCE ESCAPE PROBABILITY .
C
C       FUEL ROD PROPERTIES .
C
C      50       RF          RADIUS   .                               FT .
C               PC          DISTANCE BETWEEN RODS .                  FT.
C               CPFX        SPECIFIC HEAT .                         BTU/LB-F
C               FDENS       DENSITY .                               LB/FT**3
C               FH          LENGTH .                                 FT.
C               NA          NUMBER OF FUEL ASSEMBLIES .
C               NFRPA       FUEL RODS PER ASSEMBLY .
C
C      60       V           VELOCITY OF MODERATOR/COOLANT           FT/SEC.
C               TMI         INLET TEMPERATURE OF MODERATOR .         F.
```

```
      C
      C   70      TE          END TIME OF CALCULATION.              SEC.
      C           DH          TIME INCREMENT , THE RANGE WILL
      C                       BE BETWEEN 0.005 TO 0.05 SECOND .
      C           DELTA       CONVERGING FACTOR .
      C
      C   80      PPW         GIVEN EQUILIBRIUM POWER AT T=0.0      MW
      C           TGUES       COOLANT/MODERATOR GUESSED TEMPERATURE. F.
      C                       CORRESPONDING TO PP .
      C           TFG         FUEL GUESSED TEMPERATURE CORRESPDN  -  F.
      C                       DING TO PP .
      C
      C               MAIN PARAMETERS OTHER THAN INPUT .
      C
      C
      C   POWER DENSITY .                          PHI(1),MPE,MPEL
      C   PRECURSOR DENSITY .                      PHI(2),MQU
      C   FUEL TEMPERATURE .                       PHI(3),MTEEF
      C   MODERATOR/COOLANT TEMPERATURE .          PHI(4),MTEEM
      C   REACTIVITY AT ANY INSTANT. .             RR,RY
      C   TIME ELAPSED .                           DMH,MX
      C   TOTAL NUMBER OF ITERATIONS .             NN
      C   COOLANT/MODERATOR EQUILIBRIUM
      C                TEMPERATURE .               TMO
      C   FUEL EQUILIBRIUM TEMPERATURE .           TFO
      C
      C   HEAT TRANSFER COEFFICIENT .              HX,HGUES
      C   TOTAL MASS FLOW                          DOTMG
      C   DUMMY INDICATOR FOR PRINT-OUT THE
      C     EQUILIBRIUM CONDITION BEFORE AND
      C     AFTER REACTIVITY INSERTION .           NZ
      C   FUEL TEMPERATURE COEFFICIENT .           AAF
      C   NUMBER OF ITERATION AT ANY TIME .        N
      C
      C
      C   -------------------------------------------------------------------
      C
      C
 1        REAL MPEL(500),MPE(500),MQU(500),MTEEF(500),MTEEM(500),MX(500),RY(
         1500)
 2        DIMENSION AMX(4,4),HH(4,4),RX(4,4)
 3        DIMENSION ROOTI(4),EIGNV(4),COF(5),AX(5)
 4        DIMENSION PHI(4),BE(4),RB(4),HP(4)
 5        COMMON   BETA,XL,X,FX,RF,DH,FDENS,CPF,VF,FH,VM,DX,DOTMG,WO,HX,RR,C
         1PMX,PI,NROD
 6        DOUBLE PRECISION AX,COF,EIGNV,ROOTI
      C
      C   --------DEFINE REYNOLDS NUMBER , PRANDTL NUMBER , HEAT TRANSFER CO
      C      EFFICIENTS , FUEL COEFFICIENT TEMPERATURE AND REACTIVITY .
      C
 7        RE(RE1,RE2,RE3,RE4)=RE1*RE2*RE3/RE4
 8        PR(PR1,PR2,PR3)=PR1*PR2/PR3
 9        H(H1,H2,H3,H4,H5)=H1*H2*(H3**.8)*(H4**.3333)/H5
10        AF(AF1,AF2,AF3)=-AF1*ALOG(1.0/AF3)/SQRT(AF2)
11        RT(RT1,RT2,RT3,RT4,RT5,RT6,RT7)=RT1+(RT2+RT3)*RT4-RT5*RT2*RT6+RT7
      C
      C   ---------FUEL AND WATER PROPERTY AS TEMPERATURE DEPENDENT .
      C
12        FK(T)=0.61021E1-0.46365E-2*T+0.13063E-5*T**2
13        CK(T)=0.11711+0.13910E-2*T-0.18102E-5*T**2
```

```
      RHO(T)=0.57788E2+0.28018E-1*T-0.88346E-4*T**2
      U(T)=0.85067-0.17501E-2*T+0.11420E-5*T**2
      CP(T)=0.47088E1-0.15753E-1*T+0.17233E-4*T**2
C
C
C     --------- MAKE SURE THAT NN EQUAL TO DIMENSION NUMBER OF
C               THE PLOTTED VARIABLE .
C
      NN=1600
      NR=5
      NW=6
      NZ=0
      PI=3.14159
      N=1
      DM=0.0
      DH1=0.005
      DH2=0.05
      DMH=DM*3600.0
      R=0.0
      RR=0.0
      MX(1)=0.0
C
C     ---------READ INPUT DATA .
C
      READ (NR,29) NOPLT
      READ (NR,30) BETA,X,XL,RO,NRO,RTIME,A
      READ (NR,31) AM,G,PTFO
      READ (NR,32) RF,PC,CPFX,FDENS,FH,NA,NFRPA
      READ (NR,33) V,TMI
C
C
C     --------THE INITIAL VALUE OF DH WILL CHANGE ACCORDING TO THE LAR -
C        GEST EIGEN VALUE OF A-MATRIX .
C
C
      READ (NR,34) TE,DH,DELTA
C
C     ---------READ INITIAL POWER DESIRED AND GUESSED FUEL AND COOLANT
C        TEMPERATURE .
C
      READ (NR,35) PPW,TGUES,TFG
      PP=3.41206*PPW
C
C     ---------PRINT OUT INPUT DATA AND THE INITIAL .
C
      WRITE (NW,36)
      WRITE (NW,37)
      WRITE (NW,38) BETA,X,XL,RO
      RO=RO*BETA
      WRITE (NW,39) RTIME,A
      WRITE (NW,40) AM,G,PTFO
      WRITE (NW,41) RF,PC,CPFX,FDENS,FH
      WRITE (NW,42) NA,NFRPA
      WRITE (NW,43) V,TMI
      WRITE (NW,44) NOPLT,NRO
      WRITE (NW,45) PPW,TFG,TGUES
      WRITE (NW,46) TE,DH
      WRITE (NW,47) DELTA
C
C     --------CHANGE SECOND TO UNIT HOUR .
```

```
C
      NOH=IFIX(TE/DH+DH/2.0)
      IF (NOH.LE.NN) GO TO 1
      WRITE (NW,48)
      GO TO 28
    1 XL=XL/3600.0
      X=X*3600.
      V=V*3600.
      DH1=DH1/3600.0
      DH2=DH2/3600.0
      DH=DH/3600.
      A=A*3600.0
C
C     --------GEOMETRY CALCULATIONS FOR SQUARE PITCH.
C
      DF=2.0*RF
      FA=PI*DF*FH
      FCA=PI*DF**2/4.0
      PAREA=PC*PC
      FLA=PAREA-FCA
      NROD=NA*NFRPA
      NCHAN=NROD
      TFA=NROD*FA
      TCFA=NROD*FCA
      TCFLA=NCHAN*FLA
      VF=FCA*FH*NROD
      VM=TCFLA*FH
      PW=PI*DF
      DE=4.0*FLA/PW
      CPF=CPFX*FDENS*VF
      COL=0.042*PC/DF-0.024
C
C     --------ITERATE FUEL AND COOLANT TEMPERATURE UNTIL IT CONVERGES TO
C        THE CORRESPONDING POWER AND ITS PRECURSOR .
C
    2 CONTINUE
      UGUES=U(TGUES)
      DGUES=RHO(TGUES)
      DOTMG=DGUES*TCFLA*V
      CGUES=CK(TGUES)
      FGUES=FK(TFG)
      PKL=4.0*PI*FGUES*FH
      CPG=CP(TGUES)
      TMG=TMI+PP/(2.0*CPG*DOTMG)
      TFG=TMG+PP/(2.0*PKL*NROD)
      RN=RE(DGUES,V,DE,UGUES)
      RP=PR(CPG,UGUES,CGUES)
      HGUES=H(COL,CGUES,RN,RP,DE)
      PGUES=2.0*CPG*DOTMG*ABS(TGUES-TMI)
      DP=PGUES-PP
      DTM=TGUES-TMG
      DELPG=ABS(DP)/PP
C
C     --------SET THE GUESSED POWER AND ITERATE UNTIL CONVERGE TO THE
C        CORRESPONDING FUEL AND MODERATOR TEMPERATURE ACCURATE TO THE
C        VALUE OF DELTA .
C
      IF (DELPG-DELTA) 6,6,3
    3 IF (DTM) 4,6,5
    4 TGUES=TGUES+ABS(DTM)/2.0
```

```
      GO TO 2
    5 TGUES=TGUES-ABS(DTM)/2.0
      GO TO 2
    6 CONTINUE
      QQ=BETA*PP/(X*XL)
      QW=QQ/3.412E6
C
C     --------INITIALIZE MATRIX PHI .
C
      PHI(1)=PP
      PHI(2)=QQ
      PHI(3)=TFG
      PHI(4)=TMG
      DOTMG=DGUES*TCFLA*V
C
C     --------SETUP THE FUEL AND COOLANT TEMPERATURE AT EQUILIBRIUM
C        STATE .
C
      TMO=TMG
      TFO=TFG
      RS=RR/BETA
      WRITE (6,49)
      WRITE (NW,50) PPW,QW,TFO,TMO,RS
      WRITE (NW,51) RN,RP,COL,DOTMG,HGUES
      WRITE (NW,52)
      WRITE (NW,53)
    7 CONTINUE
      MTRX=4
C
C     --------STORE THE VALUE OF POWER , PRECURSOR DENSITY , FUEL AND MO
C        DERATOR TEMPERATURE , AND REACTIVITY FOR PLOTTING .
C
      PPW=PHI(1)/3.412E6
      QW=PHI(2)/3.412E6
      TFG=PHI(3)
      TMG=PHI(4)
      IF (NOPLT.EQ.0) GO TO 8
      MPE(N)=PPW
      MPEL(N)=ALOG10(PPW)
      MQU(N)=QW
      MTEEF(N)=TFG
      MTEEM(N)=TMG
    8 AAF=AF(G,PHI(3),PTFO)
      IF (AAF.LT.0.0.AND.AM.LT.0.0) GO TO 10
      IF (AAF+AM) 10,9,28
    9 WRITE (NW,54)
   10 UX=U(PHI(4))
      DX=RHO(PHI(4))
      CX=CK(PHI(4))
      CPMX=CP(PHI(4))
      FX=FK(PHI(3))
      DOTMG=DX*TCFLA*V
      HX=H(COL,CX,RE(DX,V,DE,UX),PR(CPMX,UX,CX),DE)
      TMOUT=2.0*PHI(4)-TMI
C
C     --------COMPUTE THE REACTIVITY INSERTED AS A FUNCTION OF TIME
C        ,AND REACTIVITY .
C
      GO TO (11,12,13,14), NRO
   11 RIN=RO
```

```
      GO TO 15
12    RIN=RO*(1+A*DM)
      GO TO 15
13    RIN=RO*COS(A*DM)
      GO TO 15
14    RIN=RO*SIN(A*DM)
15    CONTINUE
      IF (DMH-RTIME) 19,19,16
16    IF (NRO.GT.2) GO TO 19
      IF (NRO-1) 18,17,18.
17    RIN=0.0
      GO TO 19
18    RTMH=RTIME/3600.
      RIN=RO*(1+A*RTMH)
      GO TO 19
19    RY(N)=RR/BETA
C
C     ---------FORM B-MATRIX .
C
      BE(1)=0.0
      BE(2)=0.0
      BE(3)=0.0
      BE(4)=+2.0*DOTMG*TMI/(DX*VM)
C
C     ---------FORM A-MATRIX .
C
      CALL AMTRX (AMX)
C
C     ---------CHANGE A-DETERMINANT TO POLYNOMIAL FORM .
C
      CALL EIGEN4 (AMX,AX)
      M=5
      M1=M
      IF (AX(1)) 22,20,22
20    DO 21 JK=1,4
21    AX(JK)=AX(JK+1)
      MTRX=3
      M1=4
C
C     ---------COMPUTE THE EIGENVALUE OF THE A-MATRIX .
C
22    CALL POLRT (AX,COF,MTRX,EIGNV,ROOTI,IER,M1)
      MTRX=4
      ML=M1-1
C
C     ---------FIND THE LARGEST EIGENVALUE .
C
      WO=EIGNV(1)
      DO 23 IE=1,ML
      WE=EIGNV(IE)
23    WO=AMAX1(WE,WO)
C
C     ---------CALCULATE THE TIME INCREMENT , SUCH THAT THE TIME INCRE
C        MENT RECIPROCAL TO THE LARGEST EIGEN-VALUE .
C        MAKE SURE THAT THE RANGE IS BETWEEN 0.005 TO 0.05 SECOND , IF
C        SMALLER THAN 0.005 SECOND CHANGE THE VALUE TO 0.005 AND IF LAR-
C        GER THAN 0.05 SET IT TO 0.05 SECOND .
C
      DH=ABS(1.0/WO)/10.0
      DOH=DH
```

```
      IF (DDH.LT.DH1) DH=DH1
      IF (DDH.GT.DH2) DH=DH2
C
C     --------FORM R-MATRIX .
C
      CALL RMTRX (RX)
C
C     --------FORM H-MATRIX .
C
      CALL HMTRX (HH)
C
C     --------MULTIPLY R-MATRIX WITH COLUMN MATRIX-B AND MULTIPLY
C        H-MATRIX WITH COLUMN VECTOR PHI .
C
      CALL GXN (RX,BE,MTRX,RB)
      CALL GXN (HH,PHI,MTRX,HP)
C
C     --------FORM NEW PHI-MATRIX BY ADDING THE TWO COLUMN MATRIX .
C
      DO 24 IP=1,MTRX
   24 PHI(IP)=RB(IP)+HP(IP)
      WRITE (NW,55) N,DMH,RY(N),PPW,QW,TFG,TMG,TMOUT,AAF
      IF (DMH-TE) 25,27,27
C
C     --------IF MODERATOR TEMPERATURE EXCEEDING 700 DEGREE F ,PRINT A
C        WARNING AND GET CUT .
C
   25 IF (PHI(4).GE.1000.0) GO TO 26
      RR=AM*(PHI(4)-TMO)+AAF*(PHI(3)-TFO)+RIN
      R=RR
      DM=DM+DH
      DMH=DM*3600.0
      IF (DMH.GE.TE) GO TO 27
      N=N+1
      MX(N)=DMH
      GO TO 7
   26 WRITE (NW,56)
   27 NN=N
      IF (NOPLT.EQ.0) GO TO 28
C
C     --------PLOT THE RESULT SIMULTANEOUSLY IN ONE GRAPH .
      CALL TPLOT (MX,MPE,MPEL,MQU,MTEEM,MTEEF,RY,NN)
   28 STOP
C
   29 FORMAT (I2)
   30 FORMAT (2F10.5,2F10.7,I10,2F10.5)
   31 FORMAT (3F10.6)
   32 FORMAT (3F10.5,2F10.2,2I10)
   33 FORMAT (2F10.2)
   34 FORMAT (3F10.4)
   35 FORMAT (3F10.2)
   36 FORMAT (1H1)
   37 FORMAT (1X,23H***********************,/,1X,19HMODULE 2 , FEEDBACK,
     1/,1X,23H***********************,/////,1X,10HINPUT DATA,/,1X,14H***
     2***********,///)
   38 FORMAT (5X,4HBETA,5X,6HLAMBDA,5X,17HNEUTRON GEN. TIME,5X,18HINITIA
     1L REACTIVITY,/,12X,11H( SEC**-1 ),6X,7H( SEC ),18X,5H( $ ),/,3X,F8
     2.6,3X,F7.5,7X,E9.3,17X,F5.2,/)
   39 FORMAT (5X,14HINSERTION TIME,5X,15HCONSTANT PERIOD,/,9X,5H(SEC),14
     1X,7H(1/SEC),/,8X,F7.3,13X,F7.3,/)
```

```
216   40   FORMAT (5X,14HCOOLANT COEFF.,5X,10HCONSTANT B,5X,20HRESONANCE ESC.
           1 PROB.,/,6X,12H(          ),7X,8H(      ),7X,18H(                )
           2,/,6X,E12.5,7X,E8.2,13X,F5.3,/)
217   41   FORMAT (5X,11HFUEL RADIUS,5X,5HPITCH,5X,8HFUEL CP.,5X,12HFUEL DENS
           1ITY,5X,11HFUEL HEIGHT,5X,18H                  ,5X,12H            ,
           2/,7X,6H( FT ),8X,6H( FT ),2X,12H( BTU/LB-F ),3X,12H( LB/FT**3 ),7X
           3,6H( FT ),9X,16H                ,7X,10H          ,/,7X,F7.5,7X,F7.
           45,4X,F6.4,8X,F6.2,10X,F6.2,/)
218   42   FORMAT (5X,18HNUMBER OF ASSEMBLY,5X,26HNUMBER OF ROD PER ASSEMBLY,
           1/,5X,18H(                ),5X,26H(                        ),/,12X,
           2I4,23X,I4,/)
219   43   FORMAT (5X,16HCOOLANT VELOCITY,5X,20HINLET COOLANT TEMPT.,/,8X,10H
           1( FT/SEC ),15X,5H( F ),/,11X,F4.1,18X,F5.1,/)
220   44   FORMAT (9X,6HOPTION,4X,4HPLOT,9X,I1,/,19X,11HTYPE INSER.,2X,I1,/)
221   45   FORMAT (5X,13HINITIAL POWER,9X,4H(MW),1X,E11.4,/,5X,19HGUESSED FUE
           1L TEMPT.,4X,3H(F),5X,F6.1,/,5X,22HGUESSED COOLANT TEMPT.,1X,3H(F),
           26X,F5.1,///,2X,17HEND OF INPUT DATA,/,2X,18(1H*),///)
222   46   FORMAT (//,5X,12H   END TIME ,3X,14HTIME INCREMENT,/,8X,F6.3,10X,F
           16.4,//)
223   47   FORMAT (5X,22H CONVERGENCE FACTOR = ,F9.5,//)
224   48   FORMAT (//,5X,118H***  CHECK DIMENSION , IFIX(TE/DH) HAS TO BE SMA
           1LLER OR EQUAL TO THE DIMENSION OF MPEL,MPE,MTEEF,MTEEM,MY,RY AND M
           2QU .,///)
225   49   FORMAT (///,5X,27HEQUILIBRIUM STATE INITIALLY,/,4X,28(1H*),///)
226   50   FORMAT (5X,5HPOWER,5X,9HPRECURSOR,5X,11HFUEL TEMPT.,5X,14HCOOLANT
           1TEMPT.,5X,10HREACTIVITY,5X,11H           ,/,3X,E10.4,2X,E10.4,6X,F
           27.2,11X,F6.2,11X,F6.3,////)
227   51   FORMAT (85(1H*),////,5X,10HREYNOLDS #,3X,9HPRANDTL #,3X,9HCOLBURN
           1#,3X,9HMASS FLOW,3X,19HHEAT TRANFER COEFF.,/,42X,9H( LB/HR ),4X,18
           2H( BTU/HR-F-FT**2 ),/,6X,F8.1,6X,F4.1,5X,F7.3,5X,E11.4,8X,F7.1,///
           3)
228   52   FORMAT (5X,2HNO,10X,4HTIME,6X,10HREACTIVITY,7X,5HPOWER,11X,9HPRECU
           1RSOR,/,4X,4H(   ),8X,5H(SEC),9X,3H($),11X,4H(MW),14X,4H(MW))
229   53   FORMAT (7X,9HFUEL TEMP,5X,9HMOD. TEMP,3X,9HEXIT TEMP,8X,11HFUEL CO
           1EFF.,/,10X,3H(F),11X,3H(F),9X,3H(F),14X,5H(1/F),//)
230   54   FORMAT (//,8X,19H... NO FEEDBACK ...,//)
231   55   FORMAT (3X,I4,8X,F7.3,7X,F5.2,7X,E11.4,7X,E11.4,/,8X,F7.2,7X,F7.2,
           15X,F7.2,9X,E11.4)
232   56   FORMAT (//,5X,39H C R I T I C A L   T E M P E R A T U R E,//)
233        END

234        SUBROUTINE RMTRX (R)
      C
      C
      C    --------FORM R-MATRIX , DIAGONAL MATRIX IN EQUATION 2.4.20
      C       SUPPORTING ROUTINE  NONE
      C
      C
235        DIMENSION R(4,4),A(4,4)
236        COMMON   BETA,XL,X,FX,RF,DH,FDENS,CPF,VF,FH,VM,DX,DOTMG,WO,HX,RR,C
           1PMX,PI,NROD
237        CALL AMTRX (A)
238        DO 3 I=1,4
239        DO 3 II=1,4
240        IF (I-II) 1,2,1
241   1    R(I,II)=0.0
242        GO TO 3
243   2    R(I,II)=(EXP(A(I,II)*DH)-1.0)/A(I,II)
244   3    CONTINUE
```

```
      RETURN
      END

      SUBROUTINE HMTRX (H)
C
C --------FORM H-MATRIX AS IN THE EQUATION 2.4.19 .
C        SUPPORTING ROUTINE  NONE
C
C
      DIMENSION H(4,4),A(4,4)
      COMMON   BETA,XL,X,FX,RF,DH,FDENS,CPF,VF,FH,VM,DX,DOTMG,WO,HX,RR,G
     1PMX,PI,NROD
      CALL AMTRX (A)
      H(1,1)=EXP(A(1,1)*DH)
      H(1,2)=A(1,2)*(EXP(WO*DH)-EXP(A(1,1)*DH))/(WO-A(1,1))
      H(1,3)=0.0
      H(1,4)=0.0
      H(2,1)=A(2,1)*(EXP(WO*DH)-EXP(A(2,2)*DH))/(WO-A(2,2))
      H(2,2)=EXP(A(2,2)*DH)
      H(2,3)=0.0
      H(2,4)=0.0
      H(3,1)=A(3,1)*(EXP(WO*DH)-EXP(A(3,3)*DH))/(WO-A(3,3))
      H(3,2)=0.0
      H(3,3)=EXP(A(3,3)*DH)
      H(3,4)=A(3,4)*(EXP(WO*DH)-EXP(A(3,3)*DH))/(WO-A(3,3))
      H(4,1)=A(4,1)*(EXP(WO*DH)-EXP(A(4,4)*DH))/(WO-A(4,4))
      H(4,2)=0.0
      H(4,3)=0.0
      H(4,4)=EXP(A(4,4)*DH)
      RETURN
      END

      SUBROUTINE POLRT (XCOF,COF,M,ROOTR,ROOTI,IER,M1)
C
C
C        SUBROUTINE POLRT
C
C        PURPOSE
C           COMPUTES THE REAL AND COMPLEX ROOTS OF A REAL POLYNOMIAL
C
C
C        USAGE
C           CALL POLRT(XCOF,COF,M,ROOTR,ROOTI,IER,M1)
C
C        DESCRIPTION OF PARAMETERS
C           XCOF -VECTOR OF M+1 COEFFICIENTS OF THE POLYNOMIAL
C                 ORDERED FROM SMALLEST TO LARGEST POWER
C           COF  -WORKING VECTOR OF LENGTH M+1
C           M    -ORDER OF POLYNOMIAL
C           ROOTR-RESULTANT VECTOR OF LENGTH M CONTAINING REAL ROOTS
C                 OF THE POLYNOMIAL
C           ROOTI-RESULTANT VECTOR OF LENGTH M CONTAINING THE
C                 CORRESPONDING IMAGINARY ROOTS OF THE POLYNOMIAL
C           IER  -ERROR CODE WHERE
C                 IER=0  NO ERROR
C                 IER=1  M LESS THAN ONE
C                 IER=2  M GREATER THAN 36
C                 IER=3  UNABLE TO DETERMINE ROOT WITH 500 INTERATIONS
C                        ON 5 STARTING VALUES
C
```

```
C             IER=4  HIGH ORDER COEFFICIENT IS ZERO.
C        M1   -NUMBER OF COEFFICIENT , M+1
C             (ADDED ARGUMENT FROM THE ORIGINAL TO GET MORE FLEXIBLE
C                                                      -DIMENSION  )
C
C     REMARKS
C        LIMITED TO 36TH ORDER POLYNOMIAL OR LESS.
C        FLOATING POINT OVERFLOW MAY OCCUR FOR HIGH ORDER
C        POLYNOMIALS BUT WILL NOT AFFECT THE ACCURACY OF THE RESULTS.
C
C     SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED
C        NONE
C
C     METHOD
C        NEWTON-RAPHSON ITERATIVE TECHNIQUE.  THE FINAL ITERATIONS
C        ON EACH ROOT ARE PERFORMED USING THE ORIGINAL POLYNOMIAL
C        RATHER THAN THE REDUCED POLYNOMIAL TO AVOID ACCUMULATED
C        ERRORS IN THE REDUCED POLYNOMIAL.
C
C
      DIMENSION XCOF(M1),COF(M1),ROOTR(M),ROOTI(M)
      DOUBLE PRECISION XO,YO,X,Y,XPR,YPR,UX,UY,V,YT,XT,U,XT2,YT2,SUMSQ,
     1 DX,DY,TEMP,ALPHA,DABS
C
C
C     --------IF A DOUBLE PRECISION VERSION OF THIS ROUTINE IS DESIRED,
C        C IN COLUMN 1 SHOULD BE REMOVED FROM THE DOUBLE PRECISION
C        STATEMENT WHICH FOLLOWS.
C
      DOUBLE PRECISION XCCF,COF,ROOTR,ROOTI
C
C     --------THE C MUST ALSO BE REMOVED FROM DOUBLE PRECISION STATEMENT
C        APPEARING IN OTHER ROUTINES USED IN CONJUNCTION WITH THIS
C        ROUTINE.
C        THE DOUBLE PRECISION VERSION MAY BE MODIFIED BY CHANGING THE
C        CONSTANT IN STATEMENT 78 TO 1.0D-12 AND IN STATEMENT 122 TO
C        1.0D-10.  THIS WILL PROVIDE HIGHER PRECISION RESULTS AT THE
C        COST OF EXECUTION TIME
C
C
      IFIT=0
      N=M
      IER=0
      IF (XCOF(N+1)) 1,4,1
    1 IF (N) 2,2,6
C
C     --------SET ERROR CODE TO 1
C
    2 IER=1
    3 RETURN
C
C     --------SET ERROR CODE TO 4
C
    4 IER=4
      GO TO 3
C
C     --------SET ERROR CODE TO 2
C
    5 IER=2
      GO TO 3
```

```
 6     IF (N-36) 7,7,5
 7     NX=N
       NXX=N+1
       N2=1
       KJ1=N+1
       DO 8 L=1,KJ1
       MT=KJ1-L+1
 8     COF(MT)=XCOF(L)
 C
 C     ---------SET INITIAL VALUES
 C
 9     XO=.00500101
       YO=0.01000101
 C
 C     ---------ZERO INITIAL VALUE COUNTER
 C
       IN=0
 10    X=XO
 C
 C     ---------INCREMENT INITIAL VALUES AND COUNTER
 C
       XO=-10.0*YO
       YO=-10.0*X
 C
 C     ---------SET X AND Y TO CURRENT VALUE
 C
       X=XO
       Y=YO
       IN=IN+1
       GO TO 12
 11    IFIT=1
       XPR=X
       YPR=Y
 C
 C     ---------EVALUATE POLYNOMIAL AND DERIVATIVES
 C
 12    ICT=0
 13    UX=0.0
       UY=0.0
       V=0.0
       YT=0.0
       XT=1.0
       U=COF(N+1)
       IF (U) 14,27,14
 14    DO 15 I=1,N
       L=N-I+1
       TEMP=COF(L)
       XT2=X*XT-Y*YT
       YT2=X*YT+Y*XT
       U=U+TEMP*XT2
       V=V+TEMP*YT2
       FI=I
       UX=UX+FI*XT*TEMP
       UY=UY-FI*YT*TEMP
       XT=XT2
 15    YT=YT2
       SUMSQ=UX*UX+UY*UY
       IF (SUMSQ) 16,23,16
 16    DX=(V*UY-U*UX)/SUMSQ
       X=X+DX
```

```
      DY=-(U*UY+V*UX)/SUMSQ
      Y=Y+DY
      IF (DABS(DY)+DABS(DX)-1.0D-5) 21,17,17
C
C     --------STEP ITERATION COUNTER
C
   17 ICT=ICT+1
      IF (ICT-500) 13,18,18
   18 IF (IFIT) 21,19,21
   19 IF (IN-5) 10,20,20
C
C     --------SET ERROR CODE TO 3
C
   20 IER=3
      GO TO 3
   21 DO 22 L=1,NXX
      MT=KJ1-L+1
      TEMP=XCOF(MT)
      XCOF(MT)=COF(L)
   22 COF(L)=TEMP
      ITEMP=N
      N=NX
      NX=ITEMP
      IF (IFIT) 25,11,25
   23 IF (IFIT) 24,10,24
   24 X=XPR
      Y=YPR
   25 IFIT=0
      IF (DABS(Y)-1.0D-4*DABS(X)) 28,26,26
   26 ALPHA=X+X
      SUMSQ=X*X+Y*Y
      N=N-2
      GO TO 29
   27 X=0.0
      NX=NX-1
      NXX=NXX-1
   28 Y=0.0
      SUMSQ=0.0
      ALPHA=X
      N=N-1
   29 COF(2)=COF(2)+ALPHA*COF(1)
      IF (N.EQ.0) GO TO 31
      DO 30 L=2,N
   30 COF(L+1)=COF(L+1)+ALPHA*COF(L)-SUMSQ*COF(L-1)
   31 ROOTI(N2)=Y
      ROOTR(N2)=X
      N2=N2+1
      IF (SUMSQ) 32,33,32
   32 Y=-Y
      SUMSQ=0.0
      GO TO 31
   33 IF (N) 3,3,9
      END

      SUBROUTINE TPLOT (M1,M0,M2,M5,M6,M8,M9,JX)
C
C
C     --------TPLOT IS PLOTTING SEVERAL VARIABLES IN ONE GRAPH . THE X-A
C        DOES NOT REPRESENT ANY VARIABLE , IT IS INTEGER SEQUENCES .
```

```
C         FOR NEGATIVE VALUES , THE ZERO LINE IS IN THE 0.5 LINE .
C
C         SUPPORTING ROUTINE  NONE
C
          IMPLICIT REAL*4(A-H,M-Z)
          DIMENSION M8(JX),M9(JX),M0(JX),M1(JX),M2(JX),M5(JX),M6(JX)
          DIMENSION LINE(61),INUM(9)
          INTEGER PL,MI,S5,BL,SL,S9,S0,S6,S1,S2
          READ (5,8) PL,MI,S5,BL,SL,S9,S0,S1,S2,S6
          MXY=0.0
          MIN0=0.0
          MIN5=0.0
          MIN6=0.0
          MIN2=0.0
          MIN8=0.0
          MIN9=0.0
          PHI5=0.0
          PHI6=0.0
          PHI0=0.0
          PHI2=0.0
          PHI8=0.0
          PHI9=0.0
          DO 1 I=1,JX
          IF (MIN0.GT.M0(I)) MIN0=M0(I)
          IF (MIN5.GT.M5(I)) MIN5=M5(I)
          IF (MIN6.GT.M6(I)) MIN6=M6(I)
          IF (MIN2.GT.M2(I)) MIN2=M2(I)
          IF (MIN8.GT.M8(I)) MIN8=M8(I)
          IF (MIN9.GT.M9(I)) MIN9=M9(I)
          IF (ABS(M0(I)).GT.PHI0) PHI0=ABS(M0(I))
          IF (ABS(M2(I)).GT.PHI2) PHI2=ABS(M2(I))
          IF (ABS(M8(I)).GT.PHI8) PHI8=ABS(M8(I))
          IF (ABS(M9(I)).GT.PHI9) PHI9=ABS(M9(I))
          IF (ABS(M5(I)).GT.PHI5) PHI5=ABS(M5(I))
          IF (ABS(M6(I)).GT.PHI6) PHI6=ABS(M6(I))
    1     CONTINUE
          JJ=JX
          JJ0=JJ*6+1
          JJ1=JJ+1
          WRITE (6,9)
          WRITE (6,10)
          PHI0=PHI0+ABS(MIN0)
          PHI5=PHI5+ABS(MIN5)
          PHI6=PHI6+ABS(MIN6)
          PHI2=PHI2+ABS(MIN2)
          PHI8=PHI8+ABS(MIN8)
          PHI9=PHI9+ABS(MIN9)
          DO 2 I=1,JJ
          IF (MIN0.LT.0.0) M0(I)=M0(I)+ABS(MIN0)
          IF (MIN5.LT.0.0) M5(I)=M5(I)+ABS(MIN5)
          IF (MIN6.LT.0.0) M6(I)=M6(I)+ABS(MIN6)
          IF (MIN2.LT.0.0) M2(I)=M2(I)+ABS(MIN2)
          IF (MIN8.LT.0.0) M8(I)=M8(I)+ABS(MIN8)
          IF (MIN9.LT.0.0) M9(I)=M9(I)+ABS(MIN9)
          M0(I)=M0(I)/PHI0
          M5(I)=M5(I)/PHI5
          M6(I)=M6(I)/PHI6
          M2(I)=M2(I)/PHI2
          M8(I)=M8(I)/PHI8
    2     M9(I)=M9(I)/PHI9
```

```
      DO 3 I=1,9
    3 INUM(I)=I
      WRITE (6,11) (INUM(I),I=1,9)
      DO 7 I=1,JJ1
      IF (I.EQ.1) GO TO 5
      MXY=M1(I-1)
      IP8=M8(I-1)*60+1.0
      IP5=M5(I-1)*60+1.0
      IP9=M9(I-1)*60+1.0
      IP6=M6(I-1)*60+1.0
      IP0=M0(I-1)*60+1.0
      IP2=M2(I-1)*60+1.0
      DO 4 I1=1,56,5
      LINE(I1)=BL
      DO 4 I2=1,4
      I3=I1+I2
      IF (I1.EQ.IP0) LINE(I1)=S0
      IF (I3.EQ.IP0) LINE(I3)=S0
      IF (I1.EQ.IP2) LINE(I1)=S2
      IF (I3.EQ.IP2) LINE(I3)=S2
      IF (I1.EQ.IP5) LINE(I1)=S5
      IF (I3.EQ.IP5) LINE(I3)=S5
      IF (I1.EQ.IP6) LINE(I1)=S6
      IF (I3.EQ.IP6) LINE(I3)=S6
      IF (I1.EQ.IP8) LINE(I1)=SL
      IF (I3.EQ.IP8) LINE(I3)=SL
      IF (I1.EQ.IP9) LINE(I1)=S9
      IF (I3.EQ.IP9) LINE(I3)=S9
    4 CONTINUE
      LINE(61)=PL
      IIL=I-1
      IF (1.EQ.IP0) LINE(1)=S0
      IF (1.EQ.IP2) LINE(1)=S2
      IF (1.EQ.IP5) LINE(1)=S5
      IF (1.EQ.IP6) LINE(1)=S6
      IF (1.EQ.IP8) LINE(1)=SL
      IF (1.EQ.IP9) LINE(1)=S9
      IF (IP0.EQ.61) LINE(61)=S0
      IF (IP5.EQ.61) LINE(61)=S5
      IF (IP2.EQ.61) LINE(61)=S2
      IF (IP6.EQ.61) LINE(61)=S6
      IF (IP8.EQ.61) LINE(61)=SL
      IF (IP9.EQ.61) LINE(61)=S9
      IF (IP0.NE.1.OR.IP2.NE.1.OR.IP5.NE.1.OR.IP6.NE.1.OR.IP8.NE.1.OR.IP
     19.NE.1) LINE(1)=PL
      WRITE (6,12) MXY,(LINE(KK),KK=1,61)
      IF (I.EQ.JJ1) GO TO 7
    5 CONTINUE
      DO 6 I1=1,56,5
      DO 6 I2=1,4
      I3=I1+I2
      LINE(I3)=BL
    6 CONTINUE
    7 CONTINUE
      WRITE (6,13) (INUM(I),I=1,9)
      WRITE (6,15) PL,MI,S5,BL,SL,S9,S0,S1,S2,S6
      WRITE (6,16)
      WRITE (6,14)
      STOP
C
```

```
491   8    FORMAT (11A1)
492   9    FORMAT (1H1)
493  10    FORMAT (35X,16HRELATIVE DENSITY)
494  11    FORMAT (17X,9(2X,2HO.,I1,1H )/,14X,10(6H+-----),1H+)
495  12    FORMAT (1X,4HTIME,1X,F7.3,1X,61A1)
496  13    FORMAT (14X,10(6H+-----),1H+,/,16X,9(3X,2HO.,I1))
497  14    FORMAT (1H1)
498  15    FORMAT (15X,17HINPUT CHARACTER   ,11A1,/)
499  16    FORMAT (26H                         ,/,15X,10H P - POWER,/,15X,18
          1H L - LOG. OF.POWER,/,15X,22H Q - PRECURSOR DENSITY,/,15X,21H F -
          2FUEL TEMPERATURE,/,15X,26H M - MODERATOR TEMPERATURE,/,15X,15H R -
          3 REACTIVITY,/,15X,42HIF VARIABLE PLOTTED HAS NEGATIVE VALUE THE,/,
          415X,41HAXIS FOR THIS VARIABLE IS SHIFTED TO THE ,/,15X,16HCENTER O
          5F Y-AXIS,///)
500        END

501        SUBROUTINE GXN (G,XN,N,GX)
      C
      C
      C    ---------------MULTIPLY  THE G - MATRIX WITH THE INITIAL VECTOR CO
      C            LUMN TO GET THE NEXT ITERATION .
      C
      C       SUPPORTING ROUTINE, NONE
      C
      C
502        DIMENSION G(N,N),XN(N),GX(N)
503        DO 2 I=1,N
504        GG=0.0
505        DO 1 J=1,N
506   1    GG=GG+G(I,J)*XN(J)
507   2    GX(I)=GG
508        RETURN
509        END

510        SUBROUTINE AMTRX (A)
      C
      C
      C    ---------FORM A-MATRIX IN EQUATION 2.4.12 .
      C       SUPPORTING ROUTINE  NONE
      C
      C
511        DIMENSION A(4,4)
512        COMMON   BETA,XL,X,FX,RF,DH,FDENS,CPF,VF,FH,VM,DX,DOTMG,WO,HX,RR,C
          1PMX,PI,NROD
513        A(1,1)=(RR-BETA)/XL
514        A(1,2)=X
515        A(1,3)=0.0
516        A(1,4)=0.0
517        A(2,1)=BETA/XL
518        A(2,2)=-X
519        A(2,3)=0.0
520        A(2,4)=0.0
521        A(3,1)=1.0/(2.0*CPF)
522        A(3,2)=0.0
523        A(3,3)=-4.0*PI*FX*FH*NROD/CPF
524        A(3,4)=-A(3,3)
525        A(4,1)=1.0/(DX*CPMX*VM)
526        A(4,2)=0.0
```

```
      A(4,3)=0.0
      A(4,4)=-2.0*DOTMG/(DX*VM)
      RETURN
      END

      SUBROUTINE EIGEN4 (A,B)
C
C
C     --------EIGEN4 IS SUBSTITUTING DETERMINANT A TO POLYNOMIAL FORM .
C        SUPPORTING ROUTINE  NONE
C
C
      DIMENSION A(4,4),B(5)
      DOUBLE PRECISION B
      B(1)=A(1,1)*A(2,2)*A(3,3)*A(4,4)-A(1,2)*A(2,1)*A(3,3)*A(4,4)
      B(2)=-(A(1,1)*A(2,2)*A(3,3)+A(1,1)*A(2,2)*A(4,4)+A(1,1)*A(3,3)*A(4
     1,4)+A(2,2)*A(3,3)*A(4,4)-A(1,2)*A(2,1)*A(3,3)-A(1,2)*A(2,1)*A(4,4)
     2)
      B(3)=A(3,3)*A(4,4)+A(1,1)*A(2,2)+A(1,1)*A(3,3)+A(1,1)*A(4,4)+A(2,2
     1)*A(3,3)+A(2,2)*A(4,4)-A(1,2)*A(2,1)
      B(4)=-(A(1,1)+A(2,2)+A(3,3)+A(4,4))
      B(5)=1.0
      RETURN
      END

//DATA
```

```
***********************
MODULE 2 , FEEDBACK
***********************


INPUT DATA
**************


   BETA      LAMBDA      NEUTRON GEN. TIME       INITIAL REACTIVITY
           ( SEC**-1 )       ( SEC )                   ( $ )
 0.006450   0.07695       0.100E-03                     0.30

   INSERTION TIME      CONSTANT PERIOD
       (SEC)              (1/SEC)
       1.000               1.500

   COOLANT COEFF.      CONSTANT B      RESONANCE ESC. PROB.
   (           )       (        )      (                  )
   -0.50000E-04        ********             0.800

   FUEL RADIUS       PITCH      FUEL CP.       FUEL DENSITY      FUEL HEIGHT
     ( FT )         ( FT )   ( BTU/LB-F )   ( LB/FT**3 )        ( FT )
    0.01504        0.04733     0.0590          43.20             12.00

   NUMBER OF ASSEMBLY      NUMBER OF ROD PER ASSEMBLY
   (                )      (                        )
          145                         208

   COOLANT VELOCITY      INLET COOLANT TEMPT.
      ( FT/SEC )                ( F )
         13.0                   400.0

       OPTION      PLOT           1
                   TYPE INSER.    1

   INITIAL POWER           (MW)   0.1000E 04
   GUESSED FUEL TEMPT.     (F)        500.0
   GUESSED COOLANT TEMPT.  (F)        200.0

 END OF INPUT DATA
 *****************



      END TIME     TIME INCREMENT
       8.000          0.0100

    CONVERGENCE FACTOR =    0.01000
```

EQUILIBRIUM STATE INITIALLY
****

POWER	PRECURSOR	FUEL TEMPT.	COOLANT TEMPT.	REACTIVITY
0.1000E 04	0.8382E 06	504.31	412.73	0.000

****

REYNOLDS #	PRANDTL #	COLBURN #	MASS FLOW (LB/HR)	HEAT TRANFER COEFF. (BTU/HR-F-FT**2)
509503.6	1.0	0.042	0.1172E 09	9044.7

NO ( )	TIME (SEC)	REACTIVITY ($)	POWER (MW)	PRECURSOR (MW)	FUEL TEMP (F)	MOD. TEMP (F)	EXIT TEMP (F)	FUEL COEFF. (1/E)
1	0.000	0.00	0.1000E 04	0.8382E 06	504.31	412.73	425.47	0.4968E-06
2	0.046	0.30	0.9345E 03	0.8381E 06	496.42	413.04	426.08	0.5008E-06
3	0.096	0.30	0.1373E 04	0.8379E 06	497.10	412.92	425.84	0.5004E-06
4	0.146	0.30	0.1420E 04	0.8391E 06	510.64	413.39	426.79	0.4937E-06
5	0.196	0.29	0.1422E 04	0.8404E 06	521.65	413.88	427.76	0.4885E-06
6	0.246	0.29	0.1419E 04	0.8418E 06	529.63	414.32	428.64	0.4848E-06
7	0.296	0.29	0.1416E 04	0.8431E 06	535.33	414.71	429.42	0.4822E-06
8	0.346	0.28	0.1413E 04	0.8444E 06	539.41	415.06	430.12	0.4804E-06
9	0.396	0.28	0.1410E 04	0.8457E 06	542.34	415.37	430.74	0.4791E-06
10	0.446	0.28	0.1408E 04	0.8470E 06	544.44	415.65	431.30	0.4782E-06
11	0.496	0.28	0.1406E 04	0.8483E 06	545.97	415.90	431.79	0.4775E-06
12	0.546	0.28	0.1404E 04	0.8496E 06	547.09	416.12	432.23	0.4770E-06
13	0.596	0.28	0.1403E 04	0.8508E 06	547.91	416.31	432.62	0.4766E-06
14	0.646	0.27	0.1403E 04	0.8521E 06	548.53	416.49	432.97	0.4764E-06
15	0.696	0.27	0.1402E 04	0.8533E 06	549.01	416.64	433.29	0.4762E-06
16	0.746	0.27	0.1402E 04	0.8546E 06				

	549.38		416.78		433.57		0.4760E-06	
17		0.796		0.27		0.1402E 04		0.8558E 06
	549.69		416.91		433.82		0.4759E-06	
18		0.846		0.27		0.1402E 04		0.8570E 06
	549.94		417.02		434.05		0.4758E-06	
19		0.896		0.27		0.1402E 04		0.8583E 06
	550.17		417.13		434.26		0.4757E-06	
20		0.946		0.27		0.1403E 04		0.8595E 06
	550.37		417.22		434.44		0.4756E-06	
21		0.996		0.27		0.1403E 04		0.8607E 06
	550.56		417.31		434.61		0.4755E-06	
22		1.046		0.27		0.1404E 04		0.8619E 06
	550.74		417.38		434.76		0.4754E-06	
23		1.096		-0.03		0.1405E 04		0.8631E 06
	550.91		417.45		434.90		0.4753E-06	
24		1.146		-0.03		0.1011E 04		0.8643E 06
	550.95		417.52		435.03		0.4753E-06	
25		1.196		-0.03		0.9980E 03		0.8643E 06
	539.15		417.05		434.10		0.4805E-06	
26		1.246		-0.03		0.1000E 04		0.8642E 06
	530.19		416.62		433.23		0.4845E-06	
27		1.296		-0.03		0.1003E 04		0.8641E 06
	523.75		416.23		432.46		0.4875E-06	
28		1.346		-0.02		0.1005E 04		0.8640E 06
	519.15		415.88		431.77		0.4897E-06	
29		1.396		-0.02		0.1007E 04		0.8639E 06
	515.87		415.58		431.15		0.4912E-06	
30		1.446		-0.02		0.1009E 04		0.8638E 06
	513.54		415.30		430.60		0.4923E-06	
31		1.496		-0.02		0.1011E 04		0.8637E 06
	511.87		415.06		430.11		0.4931E-06	
32		1.546		-0.02		0.1013E 04		0.8637E 06
	510.67		414.84		429.68		0.4937E-06	
33		1.596		-0.01		0.1014E 04		0.8636E 06
	509.82		414.64		429.29		0.4941E-06	
34		1.646		-0.01		0.1016E 04		0.8636E 06
	509.20		414.47		428.95		0.4944E-06	
35		1.696		-0.01		0.1017E 04		0.8635E 06
	508.76		414.32		428.64		0.4946E-06	
36		1.746		-0.01		0.1018E 04		0.8635E 06
	508.43		414.18		428.37		0.4948E-06	
37		1.796		-0.01		0.1019E 04		0.8634E 06
	508.20		414.06		428.12		0.4949E-06	
38		1.846		-0.01		0.1020E 04		0.8634E 06
	508.02		413.95		427.91		0.4950E-06	
39		1.896		-0.01		0.1021E 04		0.8634E 06
	507.89		413.86		427.72		0.4951E-06	
40		1.946		-0.01		0.1021E 04		0.8633E 06
	507.79		413.77		427.55		0.4951E-06	
41		1.996		-0.01		0.1022E 04		0.8633E 06
	507.71		413.70		427.39		0.4952E-06	
42		2.046		-0.01		0.1023E 04		0.8633E 06
	507.66		413.63		427.26		0.4952E-06	
43		2.096		-0.01		0.1023E 04		0.8632E 06
	507.61		413.57		427.14		0.4952E-06	
44		2.146		-0.01		0.1023E 04		0.8632E 06
	507.57		413.52		427.03		0.4952E-06	
45		2.196		-0.01		0.1024E 04		0.8632E 06
	507.54		413.47		426.94		0.4952E-06	
46		2.246		-0.01		0.1024E 04		0.8632E 06

	507.52		413.43		426.85		0.4953E-06	
47		2.296		-0.00		0.1025E 04		0.8632E 06
	507.50		413.39		426.78		0.4953E-06	
48		2.346		-0.00		0.1025E 04		0.8631E 06
	507.48		413.35		426.71		0.4953E-06	
49		2.396		-0.00		0.1025E 04		0.8631E 06
	507.47		413.33		426.65		0.4953E-06	
50		2.446		-0.00		0.1025E 04		0.8631E 06
	507.46		413.30		426.60		0.4953E-06	
51		2.496		-0.00		0.1025E 04		0.8631E 06
	507.45		413.28		426.55		0.4953E-06	
52		2.546		-0.00		0.1026E 04		0.8631E 06
	507.44		413.25		426.51		0.4953E-06	
53		2.596		-0.00		0.1026E 04		0.8631E 06
	507.43		413.24		426.47		0.4953E-06	
54		2.646		-0.00		0.1026E 04		0.8631E 06
	507.42		413.22		426.44		0.4953E-06	
55		2.696		-0.00		0.1026E 04		0.8631E 06
	507.41		413.20		426.41		0.4953E-06	
56		2.746		-0.00		0.1026E 04		0.8630E 06
	507.41		413.19		426.38		0.4953E-06	
57		2.796		-0.00		0.1026E 04		0.8630E 06
	507.40		413.18		426.36		0.4953E-06	
58		2.846		-0.00		0.1026E 04		0.8630E 06
	507.40		413.17		426.34		0.4953E-06	
59		2.896		-0.00		0.1026E 04		0.8630E 06
	507.39		413.16		426.32		0.4953E-06	
60		2.946		-0.00		0.1026E 04		0.8630E 06
	507.39		413.15		426.30		0.4953E-06	
61		2.996		-0.00		0.1026E 04		0.8630E 06
	507.39		413.14		426.29		0.4953E-06	
62		3.046		-0.00		0.1027E 04		0.8630E 06
	507.38		413.14		426.28		0.4953E-06	
63		3.096		-0.00		0.1027E 04		0.8630E 06
	507.38		413.13		426.26		0.4953E-06	
64		3.146		-0.00		0.1027E 04		0.8630E 06
	507.38		413.13		426.25		0.4953E-06	
65		3.196		-0.00		0.1027E 04		0.8629E 06
	507.38		413.12		426.24		0.4953E-06	
66		3.246		-0.00		0.1027E 04		0.8629E 06
	507.37		413.12		426.24		0.4953E-06	
67		3.296		-0.00		0.1027E 04		0.8629E 06
	507.37		413.11		426.23		0.4953E-06	
68		3.346		-0.00		0.1027E 04		0.8629E 06
	507.37		413.11		426.22		0.4953E-06	
69		3.396		-0.00		0.1027E 04		0.8629E 06
	507.37		413.11		426.22		0.4953E-06	
70		3.446		-0.00		0.1027E 04		0.8629E 06
	507.36		413.11		426.21		0.4953E-06	
71		3.496		-0.00		0.1027E 04		0.8629E 06
	507.36		413.10		426.21		0.4953E-06	
72		3.546		-0.00		0.1027E 04		0.8629E 06
	507.36		413.10		426.20		0.4953E-06	
73		3.596		-0.00		0.1027E 04		0.8629E 06
	507.36		413.10		426.20		0.4953E-06	
74		3.646		-0.00		0.1027E 04		0.8629E 06
	507.36		413.10		426.20		0.4953E-06	
75		3.696		-0.00		0.1027E 04		0.8629E 06
	507.36		413.10		426.19		0.4953E-06	
76		3.746		-0.00		0.1027E 04		0.8628E 06

77	507.35	3.796	413.10	-0.00	426.19	0.1027E 04	0.4953E-06	0.8628E 06
78	507.35	3.846	413.09	-0.00	426.19	0.1027E 04	0.4953E-06	0.8628E 06
79	507.35	3.896	413.09	-0.00	426.19	0.1027E 04	0.4953E-06	0.8628E 06
80	507.35	3.946	413.09	-0.00	426.18	0.1027E 04	0.4953E-06	0.8628E 06
81	507.35	3.996	413.09	-0.00	426.18	0.1027E 04	0.4953E-06	0.8628E 06
82	507.35	4.046	413.09	-0.00	426.18	0.1027E 04	0.4953E-06	0.8628E 06
83	507.35	4.096	413.09	-0.00	426.18	0.1027E 04	0.4953E-06	0.8628E 06
84	507.34	4.146	413.09	-0.00	426.18	0.1027E 04	0.4953E-06	0.8628E 06
85	507.34	4.196	413.09	-0.00	426.18	0.1027E 04	0.4953E-06	0.8628E 06
86	507.34	4.246	413.09	-0.00	426.17	0.1027E 04	0.4953E-06	0.8628E 06
87	507.34	4.296	413.09	-0.00	426.17	0.1027E 04	0.4953E-06	0.8627E 06
88	507.34	4.346	413.09	-0.00	426.17	0.1027E 04	0.4953E-06	0.8627E 06
89	507.34	4.396	413.09	-0.00	426.17	0.1027E 04	0.4953E-06	0.8627E 06
90	507.34	4.446	413.09	-0.00	426.17	0.1027E 04	0.4953E-06	0.8627E 06
91	507.34	4.496	413.09	-0.00	426.17	0.1027E 04	0.4953E-06	0.8627E 06
92	507.34	4.546	413.09	-0.00	426.17	0.1027E 04	0.4953E-06	0.8627E 06
93	507.33	4.596	413.09	-0.00	426.17	0.1027E 04	0.4953E-06	0.8627E 06
94	507.33	4.646	413.08	-0.00	426.17	0.1027E 04	0.4953E-06	0.8627E 06
95	507.33	4.696	413.08	-0.00	426.17	0.1027E 04	0.4953E-06	0.8627E 06
96	507.33	4.746	413.08	-0.00	426.17	0.1027E 04	0.4953E-06	0.8627E 06
97	507.33	4.796	413.08	-0.00	426.17	0.1027E 04	0.4953E-06	0.8627E 06
98	507.33	4.846	413.08	-0.00	426.17	0.1027E 04	0.4953E-06	0.8627E 06
99	507.33	4.896	413.08	-0.00	426.17	0.1027E 04	0.4953E-06	0.8626E 06
100	507.33	4.946	413.08	-0.00	426.17	0.1027E 04	0.4953E-06	0.8626E 06
101	507.33	4.996	413.08	-0.00	426.17	0.1027E 04	0.4953E-06	0.8626E 06
102	507.32	5.046	413.08	-0.00	426.17	0.1027E 04	0.4953E-06	0.8626E 06
103	507.32	5.096	413.08	-0.00	426.17	0.1027E 04	0.4953E-06	0.8626E 06
104	507.32	5.146	413.08	-0.00	426.17	0.1027E 04	0.4953E-06	0.8626E 06
105	507.32	5.196	413.08	-0.00	426.17	0.1027E 04	0.4953E-06	0.8626E 06
106	507.32	5.246	413.08	-0.00	426.17	0.1027E 04	0.4953E-06	0.8626E 06

	507.32		413.08		426.17		0.4954E-06	
107		5.296		-0.00		0.1027E 04		0.8626E 06
	507.32		413.08		426.17		0.4954E-06	
108		5.346		-0.00		0.1027E 04		0.8626E 06
	507.32		413.08		426.17		0.4954E-06	
109		5.396		-0.00		0.1027E 04		0.8626E 06
	507.32		413.08		426.17		0.4954E-06	
110		5.446		-0.00		0.1027E 04		0.8625E 06
	507.32		413.08		426.17		0.4954E-06	
111		5.496		-0.00		0.1026E 04		0.8625E 06
	507.31		413.08		426.17		0.4954E-06	
112		5.546		-0.00		0.1026E 04		0.8625E 06
	507.31		413.08		426.17		0.4954E-06	
113		5.596		-0.00		0.1026E 04		0.8625E 06
	507.31		413.08		426.16		0.4954E-06	
114		5.646		-0.00		0.1026E 04		0.8625E 06
	507.31		413.08		426.16		0.4954E-06	
115		5.696		-0.00		0.1026E 04		0.8625E 06
	507.31		413.08		426.16		0.4954E-06	
116		5.746		-0.00		0.1026E 04		0.8625E 06
	507.31		413.08		426.16		0.4954E-06	
117		5.796		-0.00		0.1026E 04		0.8625E 06
	507.31		413.08		426.16		0.4954E-06	
118		5.846		-0.00		0.1026E 04		0.8625E 06
	507.31		413.08		426.16		0.4954E-06	
119		5.896		-0.00		0.1026E 04		0.8625E 06
	507.31		413.08		426.16		0.4954E-06	
120		5.946		-0.00		0.1026E 04		0.8625E 06
	507.30		413.08		426.16		0.4954E-06	
121		5.996		-0.00		0.1026E 04		0.8625E 06
	507.30		413.08		426.16		0.4954E-06	
122		6.046		-0.00		0.1026E 04		0.8624E 06
	507.30		413.08		426.16		0.4954E-06	
123		6.096		-0.00		0.1026E 04		0.8624E 06
	507.30		413.08		426.16		0.4954E-06	
124		6.146		-0.00		0.1026E 04		0.8624E 06
	507.30		413.08		426.16		0.4954E-06	
125		6.196		-0.00		0.1026E 04		0.8624E 06
	507.30		413.08		426.16		0.4954E-06	
126		6.246		-0.00		0.1026E 04		0.8624E 06
	507.30		413.08		426.16		0.4954E-06	
127		6.296		-0.00		0.1026E 04		0.8624E 06
	507.30		413.08		426.16		0.4954E-06	
128		6.346		-0.00		0.1026E 04		0.8624E 06
	507.30		413.08		426.16		0.4954E-06	
129		6.396		-0.00		0.1026E 04		0.8624E 06
	507.30		413.08		426.16		0.4954E-06	
130		6.446		-0.00		0.1026E 04		0.8624E 06
	507.29		413.08		426.16		0.4954E-06	
131		6.496		-0.00		0.1026E 04		0.8624E 06
	507.29		413.08		426.16		0.4954E-06	
132		6.546		-0.00		0.1026E 04		0.8624E 06
	507.29		413.08		426.16		0.4954E-06	
133		6.596		-0.00		0.1026E 04		0.8624E 06
	507.29		413.08		426.16		0.4954E-06	
134		6.646		-0.00		0.1026E 04		0.8623E 06
	507.29		413.08		426.16		0.4954E-06	
135		6.696		-0.00		0.1026E 04		0.8623E 06
	507.29		413.08		426.16		0.4954E-06	
136		6.746		-0.00		0.1026E 04		0.8623E 06

	507.29		413.08		426.16		0.4954E-06	
137		6.796		-0.00		0.1026E 04		0.8623E 06
	507.29		413.08		426.16		0.4954E-06	
138		6.846		-0.00		0.1026E 04		0.8623E 06
	507.29		413.08		426.16		0.4954E-06	
139		6.896		-0.00		0.1026E 04		0.8623E 06
	507.28		413.08		426.16		0.4954E-06	
140		6.946		-0.00		0.1026E 04		0.8623E 06
	507.28		413.08		426.16		0.4954E-06	
141		6.996		-0.00		0.1026E 04		0.8623E 06
	507.28		413.08		426.16		0.4954E-06	
142		7.046		-0.00		0.1026E 04		0.8623E 06
	507.28		413.08		426.16		0.4954E-06	
143		7.096		-0.00		0.1026E 04		0.8623E 06
	507.28		413.08		426.16		0.4954E-06	
144		7.146		-0.00		0.1026E 04		0.8623E 06
	507.28		413.08		426.16		0.4954E-06	
145		7.196		-0.00		0.1026E 04		0.8623E 06
	507.28		413.08		426.16		0.4954E-06	
146		7.246		-0.00		0.1026E 04		0.8622E 06
	507.28		413.08		426.16		0.4954E-06	
147		7.296		-0.00		0.1026E 04		0.8622E 06
	507.28		413.08		426.16		0.4954E-06	
148		7.346		-0.00		0.1026E 04		0.8622E 06
	507.28		413.08		426.16		0.4954E-06	
149		7.396		-0.00		0.1026E 04		0.8622E 06
	507.27		413.08		426.16		0.4954E-06	
150		7.446		-0.00		0.1026E 04		0.8622E 06
	507.27		413.08		426.16		0.4954E-06	
151		7.496		-0.00		0.1026E 04		0.8622E 06
	507.27		413.08		426.16		0.4954E-06	
152		7.546		-0.00		0.1026E 04		0.8622E 06
	507.27		413.08		426.15		0.4954E-06	
153		7.596		-0.00		0.1026E 04		0.8622E 06
	507.27		413.08		426.15		0.4954E-06	
154		7.646		-0.00		0.1026E 04		0.8622E 06
	507.27		413.08		426.15		0.4954E-06	
155		7.696		-0.00		0.1026E 04		0.8622E 06
	507.27		413.08		426.15		0.4954E-06	
156		7.746		-0.00		0.1026E 04		0.8622E 06
	507.27		413.08		426.15		0.4954E-06	
157		7.796		-0.00		0.1026E 04		0.8622E 06
	507.27		413.08		426.15		0.4954E-06	
158		7.846		-0.00		0.1026E 04		0.8621E 06
	507.27		413.08		426.15		0.4954E-06	
159		7.896		-0.00		0.1026E 04		0.8621E 06
	507.26		413.08		426.15		0.4954E-06	
160		7.946		-0.00		0.1026E 04		0.8621E 06
	507.26		413.08		426.15		0.4954E-06	
161		7.996		-0.00		0.1026E 04		0.8621E 06
	507.26		413.07		426.15		0.4954E-06	

```
                                RELATIVE DENSITY
                 0.1   0.2   0.3   0.4   0.5   0.6   0.7   0.8   0.9
            +-----+-----+-----+-----+-----+-----+-----+-----+-----+-----+
TIME  0.000 |     R                                   P           F  LQM|
TIME  0.046 |                                      P              F L QR|
TIME  0.096 |                                                     F  PQMR
TIME  0.146 |                                                      F  QR|
TIME  0.196 |                                                       F QRL
TIME  0.246 |                                                        FRM|
TIME  0.296 |                                                         RM|
TIME  0.346 |                                                        RFM|
TIME  0.396 |                                                        RQF|
TIME  0.446 |                                                       R QF|
TIME  0.496 |                                                       R QF|
TIME  0.546 |                                                       R QF|
TIME  0.596 |                                                      R   F|
TIME  0.646 |                                                      R   F|
TIME  0.696 |                                                      R   F|
TIME  0.746 |                                                      R   F|
TIME  0.796 |                                                      R   F|
TIME  0.846 |                                                     R    F|
TIME  0.896 |                                                     R    F|
TIME  0.946 |                                                     R    F|
TIME  0.996 |                                                     R    F|
TIME  1.046 |                                                     R    F|
TIME  1.096 |                                                          F|
TIME  1.146 |                                         P              L  F
TIME  1.196 |                                         P              LFM|
TIME  1.246 |                                         P              F M|
TIME  1.296 |R                                        P              F M|
TIME  1.346 |R                                        P             FL M|
TIME  1.396 | R                                       P             FL M|
TIME  1.446 | R                                       P            F L M|
TIME  1.496 | R                                       P            F L M|
TIME  1.546 |  R                                      P            F L M|
TIME  1.596 |  R                                      P            F L M|
TIME  1.646 |  R                                      P            F L M|
TIME  1.696 |  R                                      P            F L M|
TIME  1.746 |   R                                     P            F L M|
TIME  1.796 |   R                                     P            F L M|
TIME  1.846 |   R                                      P           F L M|
TIME  1.896 |   R                                      P           F L M|
TIME  1.946 |   R                                      P           F L M|
TIME  1.996 |   R                                      P           F L M|
TIME  2.046 |   R                                      P           F L M|
TIME  2.096 |   R                                      P           F L M|
TIME  2.146 |    R                                     P           F L M|
TIME  2.196 |    R                                     P           F L M|
TIME  2.246 |    R                                     P           F L M|
TIME  2.296 |    R                                     P           F L M|
TIME  2.346 |    R                                     P           F L M|
TIME  2.396 |    R                                     P           F L M|
TIME  2.446 |    R                                     P           F L M|
TIME  2.496 |    R                                     P           F L M|
TIME  2.546 |    R                                     P           F L M|
TIME  2.596 |    R                                     P           F L M|
TIME  2.646 |    R                                     P           F L M|
TIME  2.696 |    R                                     P           F L M|
TIME  2.746 |    R                                     P           F L M|
```

TIME	2.796 |	R	P	F L M|
TIME	2.846 |	R	P	F L M|
TIME	2.896 |	R	P	F L M|
TIME	2.946 |	R	P	F L M|
TIME	2.996 |	R	P	F L M|
TIME	3.046 |	R	P	F L M|
TIME	3.096 |	R	P	F L M|
TIME	3.146 |	R	P	F L M|
TIME	3.196 |	R	P	F L M|
TIME	3.246 |	R	P	F L M|
TIME	3.296 |	R	P	F L M|
TIME	3.346 |	R	P	F L M|
TIME	3.396 |	R	P	F L M|
TIME	3.446 |	R	P	F L M|
TIME	3.496 |	R	P	F L M|
TIME	3.546 |	R	P	F L M|
TIME	3.596 |	R	P	F L M|
TIME	3.646 |	R	P	F L M|
TIME	3.696 |	R	P	F L M|
TIME	3.746 |	R	P	F L M|
TIME	3.796 |	R	P	F L M|
TIME	3.846 |	R	P	F L M|
TIME	3.896 |	R	P	F L M|
TIME	3.946 |	R	P	F L M|
TIME	3.996 |	R	P	F L M|
TIME	4.046 |	R	P	F L M|
TIME	4.096 |	R	P	F L M|
TIME	4.146 |	R	P	F L M|
TIME	4.196 |	R	P	F L M|
TIME	4.246 |	R	P	F L M|
TIME	4.296 |	R	P	F L M|
TIME	4.346 |	R	P	F L M|
TIME	4.396 |	R	P	F L M|
TIME	4.446 |	R	P	F L M|
TIME	4.496 |	R	P	F L M|
TIME	4.546 |	R	P	F L M|
TIME	4.596 |	R	P	F L M|
TIME	4.646 |	R	P	F L M|
TIME	4.696 |	R	P	F L M|
TIME	4.746 |	R	P	F L M|
TIME	4.796 |	R	P	F L M|
TIME	4.846 |	R	P	F L M|
TIME	4.896 |	R	P	F L M|
TIME	4.946 |	R	P	F L M|
TIME	4.996 |	R	P	F L M|
TIME	5.046 |	R	P	F L M|
TIME	5.096 |	R	P	F L M|
TIME	5.146 |	R	P	F L M|
TIME	5.196 |	R	P	F L M|
TIME	5.246 |	R	P	F L M|
TIME	5.296 |	R	P	F L M|
TIME	5.346 |	R	P	F L M|
TIME	5.396 |	R	P	F L M|
TIME	5.446 |	R	P	F L M|
TIME	5.496 |	R	P	F L M|
TIME	5.546 |	R	P	F L M|
TIME	5.596 |	R	P	F L M|
TIME	5.646 |	R	P	F L M|
TIME	5.696 |	R	P	F L M|
TIME	5.746 |	R	P	F L M|

```
TIME     5.796 I      R                                           P                 F L MI
TIME     5.846 I      R                                           P                 F L MI
TIME     5.896 I      R                                           P                 F L MI
TIME     5.946 I      R                                           P                 F L MI
TIME     5.996 I      R                                           P                 F L MI
TIME     6.046 I      R                                           P                 F L MI
TIME     6.096 I      R                                           P                 F L MI
TIME     6.146 I      R                                           P                 F L MI
TIME     6.196 I      R                                           P                 F L MI
TIME     6.246 I      R                                           P                 F L MI
TIME     6.296 I      R                                           P                 F L MI
TIME     6.346 I      R                                           P                 F L MI
TIME     6.396 I      R                                           P                 F L MI
TIME     6.446 I      R                                           P                 F L MI
TIME     6.496 I      R                                           P                 F L MI
TIME     6.546 I      R                                           P                 F L MI
TIME     6.596 I      R                                           P                 F L MI
TIME     6.646 I      R                                           P                 F L MI
TIME     6.696 I      R                                           P                 F L MI
TIME     6.746 I      R                                           P                 F L MI
TIME     6.796 I      R                                           P                 F L MI
TIME     6.846 I      R                                           P                 F L MI
TIME     6.896 I      R                                           P                 F L MI
TIME     6.946 I      R                                           P                 F L MI
TIME     6.996 I      R                                           P                 F L MI
TIME     7.046 I      R                                           P                 F L MI
TIME     7.096 I      R                                           P                 F L MI
TIME     7.146 I      R                                           P                 F L MI
TIME     7.196 I      R                                           P                 F L MI
TIME     7.246 I      R                                           P                 F L MI
TIME     7.296 I      R                                           P                 F L MI
TIME     7.346 I      R                                           P                 F L MI
TIME     7.396 I      R                                           P                 F L MI
TIME     7.446 I      R                                           P                 F L MI
TIME     7.496 I      R                                           P                 F L MI
TIME     7.546 I      R                                           P                 F L MI
TIME     7.596 I      R                                           P                 F L MI
TIME     7.646 I      R                                           P                 F L MI
TIME     7.696 I      R                                           P                 F L MI
TIME     7.746 I      R                                           P                 F L MI
TIME     7.796 I      R                                           P                 F L MI
TIME     7.846 I      R                                           P                 F L MI
TIME     7.896 I      R                                           P                 F L MI
TIME     7.946 I      R                                           P                 F L MI
TIME     7.996 I      R                                           P                 F L MI
               +------+------+------+------+------+------+------+------+------+------+
                     0.1    0.2    0.3    0.4    0.5    0.6    0.7    0.8    0.9
               INPUT CHARACTER  I*Q FRP1LM

                P - POWER
                L - LOG. OF POWER
                Q - PRECURSOR DENSITY
                F - FUEL TEMPERATURE
                M - MODERATOR TEMPERATURE
                R - REACTIVITY
               IF VARIABLE PLOTTED HAS NEGATIVE VALUE THE
               AXIS FOR THIS VARIABLE IS SHIFTED TO THE
               CENTER OF Y-AXIS
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